Chapter 1

Sample Pretest

Part I: SCIENTIFIC CALCULATOR REQUIRED

1. [6 points] Compute each number rounded to 3 decimal places. Please double check your answer.
   
   \begin{align*}
   (a) \quad & \frac{\sqrt{2} + 3}{\pi} \\
   (b) \quad & \sqrt{\frac{\pi^2 + 7}{1.3 + \sqrt{7}}} \\
   
   \text{Answers}
   \end{align*}

Part II: NO CALCULATORS!

2. [3 points] Write each number as a decimal.
   
   \begin{align*}
   (a) \quad & 2.1 \times 10^4 \\
   (b) \quad & 3.26 \times 10^{-5} \\
   (c) \quad & 1 \times 10^{-2} \\
   
   \text{Answers}
   \end{align*}

3. [3 points] Write each number in scientific notation.
   
   \begin{align*}
   (a) \quad & 9582 \\
   (b) \quad & 1.245 \\
   (c) \quad & .000561 \\
   
   \text{Answers}
   \end{align*}

4. [3 points] Compute the following. Simplify.
   
   \begin{align*}
   (a) \quad & -\frac{3}{4} - \frac{5}{12} \\
   (b) \quad & \sqrt[3]{-|4^2 - 2^3|} \\
   (c) \quad & \left(\frac{8}{27}\right)^{-2/3} \\
   
   \text{Answers}
   \end{align*}
CHAPTER 1. SAMPLE PRETEST

5. [10 points] Simplify each expression so that all the exponents are positive.
   (a) $h \frac{g^3 h^{-2}}{q^2 g h^3}$
   (b) $\frac{a^{-1} + b^{-1}}{a^{-1} - b^{-1}}$

6. [10 points] Rewrite each expression as a polynomial in standard form.
   (a) $(x - 2)^2(x + 2)$
   (b) $2(x + 7) - (2x - 3)^2$

7. [10 points] Factor over the reals.
   (a) $x^2 - 9x + 14$
   (b) $x^4 - 7x^2 + 12$

8. [10 points] Perform the indicated operations and simplify the result as much as possible. (Assume $x \neq 1$ or 2.)
   $$\frac{x + 1}{x - 1} + \frac{2x}{x - 2}$$

9. [15 points] Divide: $\frac{x^4 - 3x^3 + 2x^2 - x + 4}{x^2 + 3}$

10. [10 points] A right triangle has hypotenuse of length 10 feet and one leg in known to be 6 feet. What is the length of the other leg? What is the triangle’s area?

11. [10 points] An isosceles triangle has two legs of length 6 inches and one of length 4 inches. Find the triangle’s area. Hint: draw a picture.

12. [10 points] Construct a formula for the area $A$ of a circle in terms its circumference $C$. Hint: you should know the formulas for $A$ and $C$ in terms of the radius $R$. 

Answers
1.1 Answers

1. (a) 0.657 (b) 2.068

2. (a) 21,000 (b) 0.0000326 (c) 0.01

3. (a) $9.582 \times 10^3$ (b) 1.245 (c) $5.61 \times 10^{-4}$

4. (a) $-\frac{7}{6}$ (b) $-2$ (c) $\frac{9}{4}$

5. (a) $\frac{g^2}{q^2 h^4}$ (b) $\frac{b + a}{b - a}$

6. (a) $x^3 - 2x^2 - 4x + 8$ (b) $-4x^2 + 14x + 5$

7. (a) $(x - 7)(x - 2)$ (b) $(x + \sqrt{3})(x - \sqrt{3})(x + 2)(x - 2)$

8. $\frac{3x^2 - 3x - 2}{x^2 - 3x + 2}$

9. Quotient = $x^2 - 3x - 1$. Remainder = $8x + 7$.

10. Leg = 8 feet. Area = 24 square feet.

11. Area = $4\sqrt{5}$ square inches.

12. $A = \frac{C^2}{4\pi}$.
Chapter 2

Practice Finals

No final exam can cover every single course objective. The practice finals given here are meant to give the student a general sense of the format and level of difficulty of a typical final exam. Studying these may be helpful but is in no way a substitute for studying your homeworks, class tests and quizzes. Just because a certain type of problem does not appear on either of these practice finals does not mean it will not be on your final exam.

2.1 Practice Final 1

1. [20 points; 4 points each]
   a) Simplify the following completely; express your answer using only positive exponents.
   \[
   \left( \frac{27x^{-6}}{1000y^{-9}} \right)^{2/3}
   \]
   b) Factor over the reals: \(2x^4 - 7x^2 - 4\).
   c) Solve for \(x\): \(\frac{3x - 4}{x} = 2\).
   d) Solve for \(x\): \(\log_2 3x = 4\).
   e) Find the equation of the line going through (2,3) and (−1,2), in slope-intercept form.

Answers
2. [20 points; 5 points each]

   a) Find all real or complex values of \( x \) that solve, \( \frac{1 - 3x}{4} = \frac{3}{1 + 3x} \).
   
   b) Find the equation, in standard form, of the circle passing through (0, 1) with center (2, −1).
   
   c) Solve \(|2x - 3| > 5\). Write your answer in interval notation.
   
   d) Compute \( \log_7 5 \), rounded to 5 decimal places.

   Answers

3. [20 points; 10 points each]

   a) Let \( g(x) = x^3 + 1 \). Graph \( y = g(x) \). b) Graph \( y = g^{-1}(x) \) on the same grid. Label the intercepts on both graphs.

   Answers

4. [20 points; 10 points each]

   a) Sketch the graph of \( y = \ln(x - 3) \). [6 points] Label the intercepts and asymptotes. [4 points]

   b) Graph \( y = x^2 - 6x + 8 \). [5 points] Label all of the following: the intercepts, the vertex, and the axis of symmetry. [5 points]

   Answers

5. [20 points; 10 points each]

   a) List the potential rational zeros of \( 6x^2 + 7x - 3 \), according to the Rational Zeros Theorem. [5 points] Factor \( 6x^2 + 7x - 3 \). [5 points]

   b) Factor \( 2x^3 + x^2 - 5x + 2 \).

   Answers

6. [20 points; 10 points each]

   a) Solve for \( x \) where \( 8^{2x-6} = 4^{x+1} \).

   b) Solve for \( x \) where \( \log_3(x^2 + x) - \log_3(x^2 - x) = 1 \).

   Answers
7. [20 points] Let \( f(x) = \frac{x}{x^2 - 4} \).
   
   a) [2 points] State the domain of \( f \):
   
   b) [3 points] Is \( f \) even, odd or neither?
   
   c) [3 points] Find all the asymptotes for the graph of \( y = f(x) \).
   
   d) [2 points] Find all the intercepts for the graph of \( y = f(x) \).
   
   e) [10 points] Sketch the graph labeling the intercepts and asymptotes.

8. [20 points; 10 points each]
   
   a) Find a 6 degree polynomial function \( f(x) \) with real coefficients that has one zero at \( x = 0 \) with multiplicity 4, and a complex zero at \( x = 2 - i \). Express your answer in standard form.

   b) Find the equation of the parabola below, expressed in standard form.

   ![Parabola Diagram]

9. [20 points] Suppose you have 20 liters of a solution that is 30% isopropyl alcohol. How many liters of 80% alcohol would you have to add to get a solution that is 50% alcohol?
10. [20 points] A wire 10 meters long is to be cut into two pieces. Say the first piece has length $a$ and the second length $b$. Clearly $a + b = 10$. The first piece will be shaped into a square, the second into a circle.

![Diagram of wire cut into two pieces, one shaped into a square and the other into a circle]

a) [4 points] Find the area of the square as a function of $a$.

b) [6 points] Find the area of the circle as a function of $b$.

c) [2 points] Now find the area of the circle as a function of $a$; remember $a + b = 10$.

d) [4 points] Now find a formula for the total enclosed area in terms of $a$. Simplify it.

e) [4 points] This function should be a quadratic. For what value of $a$ is it a minimum?

Answers
2.2. Answers and Hints

1. [20 points; 4 points each]
   a) \( \frac{9y^6}{100x^4} \)
   b) \((2x^2 + 1)(x - 2)(x + 2)\)
   c) \(x = 4\).
   d) \(x = 16/3\)
   e) \(y = x/3 + 7/3\).

2. [20 points; 5 points each]
   a) \(\pm i\sqrt{11}/3\).
   b) \((x - 2)^2 + (y + 1)^2 = 8\).
   c) \((-\infty, -1) \cup (4, \infty)\)
   d) 0.82709

3. [20 points; 10 points each]
4. [20 points; 10 points each]
   a) \( y = \ln(x - 3) \)

   \[ x=3 \]

   \( (4,0) \)

   b) \( y = x^2 - 6x + 8 \).

   \[ (4,0) \]  \( (2,0) \)  \( (3,-1) \)  \( (0,8) \)

5. [20 points; 10 points each]
   a) \( \pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{9} \).  \( 6x^2 + 7x - 3 = (2x + 3)(3x - 1) \) or \( 6(x + \frac{3}{2})(x - \frac{1}{3}) \).
   b) \( 2x^3 + x^2 - 5x + 2 = (2x - 1)(x + 2)(x - 1) \).
6. [20 points; 10 points each]
   a) \( x = 5 \)
   b) \( x = 2 \). (\( x = 0 \) is invalid.)

7. [20 points] Let \( f(x) = \frac{x}{x^2 - 4} \)
   a) [2 points] All real numbers except 2 and \(-2\), or \((-\infty, -2) \cup (-2, 2) \cup (2, \infty)\).
   b) [3 points] Odd.
   c) [3 points] Vertical: \( x = 2, x = -2 \). Horizontal: \( y = 0 \) (i.e., the \( x \)-axis).
   d) [2 points] \((0, 0)\) is the only intercept.
   e) [10 points] Sketch the graph labeling the intercepts and asymptotes.

8. [20 points; 10 points each]
   a) \( x^6 - 4x^5 + 5x^4 \)
   b) \(-\frac{1}{10}x^2 + 10\).
9. [20 points] Set up the equation $20(0.3) + x(0.8) = (20 + x)(0.5)$. Thus, $x = 6\frac{2}{3}$.

Return to Problem

10. [20 points]
   a) [4 points] $A_S = \frac{a^2}{16}$
   b) [6 points] $A_C = \frac{b^2}{4\pi}$
   c) [2 points] $A_C = \frac{(10 - a)^2}{4\pi}$
   d) [4 points] $A = A_S + A_C = \left(\frac{1}{16} + \frac{1}{4\pi}\right)a^2 - \frac{5}{\pi}a + \frac{25}{\pi}$
   e) [4 points] $a_{\text{min}} = \frac{40}{\pi + 4}$

Return to Problem

2.3 Practice Final 2

1. [20 points; 4 points each]
   a) Simplify the following completely; express your answer using only positive exponents. (Assume both $x$ and $y$ are positive.)
   $\left(\frac{x^3y^9}{8\sqrt{x^{12}y^{-6}}}\right)^{-1/3}$
   b) Solve for $t$ in the equation $A = A_0e^{rt}$.
   c) Rationalize the denominator of $\frac{x}{\sqrt{x} - \sqrt{2}}$.
   d) A right triangle has area 3 and one leg of length 2. What is the length of its hypotenuse?
   e) Multiply: $(5 + 3i)(2 - i)$; express your answer in the form $a + bi$, where $a$ and $b$ are real numbers.

Answers
2. [20 points; 5 points each]
   a) Find the equation of the line going through (2, 3) and perpendicular to the line given by $y = 3x + 2$, in slope-intercept form.
   b) Solve for $x$ in the equation $3^x = 11$. Round your answer to five decimal places.
   c) Divide: $\frac{2 + 3i}{1 - i}$; express your answer in the form $a + bi$, where $a$ and $b$ are real numbers.
   d) Let $f(x) = 17 + \ln x^3$. Find $f^{-1}(20)$.

3. [20 points] The equation of a circle in general form is $x^2 + y^2 + 8x - 6y + 8 = 0$.
   a) [10 points] Put the equation into standard form.
   b) [4 points] What are the center and radius of the circle?
   c) [6 points] Graph the circle.

4. [20 points; 10 points each]
   a) Let $f(x) = x^3 + x^2 - 4x + 6$. Factor $f(x)$ over the complex numbers. Hint: check that $f(-3) = 0$.
   b) Find a 4 degree polynomial function $f(x)$ with real coefficients that has complex zeros at $x = 3 - 2i$ and $x = 2 + i$. Express your answer in standard form.

5. [20 points; 10 points each]
   a) Find all real values of $x$ such that $x^2 - x = 1$.
   b) Find all real values of $x$ such that $e^{2x} - e^x = 1$.
6. [20 points; 10 points each]
   a) Solve the inequity \( \frac{x + 1}{2x - 3} \geq 0 \); express your answer using interval notation.
   b) Solve the inequity \( \frac{x + 1}{2x - 3} \geq 1 \); express your answer using interval notation.

Answers

7. [20 points] Let \( f(x) = x^2 + 1 \).
   a) [5 points] Graph \( y = f(x) \).
   b) [10 points] Find a formula for the slope of the line segment joining \((2, f(2))\) with \((2 + h, f(2 + h))\) in terms of \(h\); simplify it.
   c) [5 points] What value does the slope approach when \(h\) tends toward 0?

Answers

8. [20 points] The weight of a colony of bacteria at time \(t\) in hours obeys the equation \( B(t) = B_0 e^{kt} \). The colony has an initial weight of 15 grams. In 10 hours the weight increased to 20 grams.
   a) [10 points] Find \(k\). (Round your answer to six decimal places.)
   b) [10 points] How long will it take for the colony to double its weight? (Express your answer in hours and minutes, rounded to the nearest minute.)

Answers

9. [20 points] Let \( f(x) = \frac{x + 1}{x^2 + 2x - 3} \).
   a) [2 points] State the domain of \(f\):
   b) [3 points] Is \(f\) even, odd or neither?
   c) [3 points] Find all the asymptotes for the graph of \(y = f(x)\).
   d) [2 points] Find all the intercepts for the graph of \(y = f(x)\).
   e) [10 points] Sketch the graph labeling the intercepts and asymptotes.

Answers
10. [20 points; 10 points each] A gardener has 240 feet of fencing to enclose two adjacent rectangular growing areas as pictured. Both rectangles are to have the same dimensions.

a) Express the total growing area as a function of \( x \).

b) What dimensions should be used so that the maximum growing area will be enclosed? (You must find both \( x \) and \( y \).)

2.4 Answers and Hints

1. [20 points]
   a) \( \frac{2x}{y^2} \)
   b) \( t = \frac{\ln(A/A_0)}{r} \)
   c) \( \frac{x(\sqrt{x} + \sqrt{2})}{x^2 - 2} \)
   d) \( \sqrt{13} \)
   e) 13 + 1i, but 13 + i is acceptable, even preferable.
2. [20 points]
   a) \( y = -\frac{1}{3}x + 3\frac{2}{3} \)
   b) \( x = \frac{\log 11}{\log 3} \approx 2.18266 \)
   c) \( -\frac{3}{2} + \frac{5}{2}i \)
   d) \( f^{-1}(20) = e \).

3. [20 points]
   a) \((x + 4)^2 + (y - 3)^2 = 17\)
   b) Center = \((-4, 3)\). Radius = \(\sqrt{17}\).
   c) Easy.

4. [20 points]
   a) \((x + 3)(x - (1 - i))(x - (1 + i))\)
   b) \((x^2 - 6x + 13)(x^2 - 4x + 5) = x^4 - 10x^3 + 42x^2 - 82x + 65\)

5. [20 points]
   a) \(\frac{1 \pm \sqrt{5}}{2}\)
   b) \(\ln \left( \frac{1 + \sqrt{5}}{2} \right)\). Note that \(\ln \left( \frac{1 - \sqrt{5}}{2} \right)\) is undefined (or at least is not a real number) and hence is not a valid solution.

6. [20 points]
   a) \((-\infty, -1] \cup (\frac{3}{2}, \infty)\)
   b) \((\frac{3}{2}, 4]\)
2.4. ANSWERS AND HINTS

7. [20 points]
   a) Easy.
   b) \( m(h) = 4 + h. \)
   c) \( \lim_{h \to 0} m(h) = 4. \)

8. [20 points]
   a) \( k \approx 0.028768. \)
   b) 24 hours, 6 minutes.

9. [20 points]
   a) All real numbers except \(-3\) and 1, or \((-\infty, -3) \cup (-3, 1) \cup (1, \infty).\)
   b) neither
   c) Vertical: \( x = -3 \) and \( x = 1. \) Horizontal: \( y = 0 \) (i.e., the \( x \)-axis).
   d) \( (0, -\frac{1}{3}), (-1, 0). \)
   e)

10. [20 points]
    a) \( A = -3x^2/2 + 120x. \)
    b) \( x = 40 \) and \( y = 60 \) feet.