Math 250 Algebra/Trig Pre-Test

This pre-test contains basic questions on algebra and trigonometry. Solutions are provided at the end of the test along with references to where you can go for additional review and practice problems. It is important that you work through any necessary review. When completed, move on to the Skills Assessment Test and begin your preparation for Calculus II.

You may click on the blue words if you wish to jump to an answer or the review topics.

If you would like to print the Pre-Test so you can work it out on paper, please click Print.

1. Factor completely: \(2x^4 - 5x^3 - 3x^2\).  
   Answer

2. Find the quotient and remainder for \(\frac{x^3 - 2x - 3}{x - 2}\).  
   Answer

3. Expand \(e^x(1 + e^{2x})^2\).  
   Answer

4. Complete the square given \(x^2 - 6x - 2\).  
   Answer

5. Find the interval on which \(|x - 3| < 1\).  
   Answer

6. Consider the function \(f(x) = 2x - x^2\). Find and simplify \(\frac{f(x + h) - f(x)}{h}\).  
   Answer

7. Consider the function: \(f(x) = \frac{x}{2^x}\). Find and simplify \(\frac{f(n + 1)}{f(n)}\).  
   Answer
8. Fill in the blanks.
   
   a) As $x \to -\infty$, $e^x \to \underline{\hspace{2cm}}$.
   
   b) As $t \to 0$, $60(1 - e^{2t}) \to \underline{\hspace{2cm}}$.
   
   c) As $x \to 0^+$, $\ln x \to \underline{\hspace{2cm}}$.
   
   d) As $n \to \infty$, $\left(1 + \frac{1}{n}\right)^n \to \underline{\hspace{2cm}}$.

The remaining problems should be done without a calculator.

9. Complete the following tables. (You know how helpful it is in Calc I to be able to evaluate the trig functions for the values of $\theta$ in the table. The same comment holds for Calc II.)

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<thead>
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<th>$\frac{\pi}{6}$</th>
<th>$\frac{\pi}{4}$</th>
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<th>$\frac{3\pi}{4}$</th>
<th>$\frac{5\pi}{6}$</th>
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<tbody>
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9. (continued)

<table>
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<th>$\frac{11\pi}{6}$</th>
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</tbody>
</table>

10. Graph one period of the functions $\sin x$, $\cos x$, and $\tan x$.  

11. Fill in the blanks.

   a) $\sin^{-1}(0) =$ __________;  
   b) $\arcsin \left(\frac{1}{2}\right) =$ __________;  
   c) $\arctan(1) =$ __________;  
   d) $\tan^{-1}(-1) =$ __________;  
   e) $\sin^{-1}(-1) =$ __________;  
   f) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) =$ __________.

12. As $x \to \frac{\pi^-}{2}$, $\tan x \to$ __________.  

   As $x \to -\frac{\pi^+}{2}$, $\tan x \to$ __________.  

   This means that:  
   As $x \to \infty$, $\tan^{-1}(x) \to$ __________.  
   As $x \to -\infty$, $\tan^{-1}(x) \to$ __________.
13. Complete the following identities.

a) \( \sin^2 \theta + \cos^2 \theta = \underline{\text{1}} \).

b) \( \tan^2 \theta + \underline{\text{1}} = \sec^2 \theta \).

c) \( \cos(a + b) = \underline{\text{0}} \).

d) \( \sin(2\theta) = \underline{\text{0}} \) (Answer in terms of \( \sin \theta \) and \( \cos \theta \)).

e) \( \sin^2 \theta = \underline{\text{0}} \) (Answer in terms of \( \cos 2\theta \)).

f) \( \cos^2 \theta = \underline{\text{0}} \) (Answer in terms of \( \cos 2\theta \)).

14. Solve the following trigonometric equations, where \( 0 \leq \theta < 2\pi \).

a) \( \cos \theta = \frac{\sqrt{3}}{2} \)

b) \( \tan \theta = -1 \)

c) \( 1 - \sin \theta - 2 \sin^2 \theta = 0 \) \hspace{1cm} \text{Answers}

15. If \( \tan \theta = \frac{x}{4} \) where \( x > 0 \), find \( \sin \theta \) and \( \cos \theta \) in terms of \( x \). \hspace{1cm} \text{Answer}
ANSWERS to PRE-TEST

1. \( x^2(2x + 1)(x - 3) \)  
   (see Math 108, Review Topic 4 for help.)

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2. \( x^2 + 2x + 2 + \frac{1}{x - 2} \)  
   (see Math 108, Review Topic 3 for help.)

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3. \( e^x + 2e^{3x} + e^{5x} \)  
   (see Math 150, Review Topic 5 for help.)

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4. \( (x^2 - 6x + 9) - 2 - 9 = (x - 3)^2 - 11 \)  
   (see Math 150, Review Topic 7 for help.)

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5. \(-1 < x - 3 < 1, \text{ or} 2 < x < 4 \)  
   (see Math 150, Review Topic 2 for help.)

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6. \( f(x + h) = 2(x + h) - (x + h)^2 \)  
   \[
   \frac{f(x + h) - f(x)}{h} = \frac{2(x + h) - (x + h)^2 - (2x - x^2)}{h} = \frac{2h - 2xh - h^2}{h} = 2 - 2x - h
   \]  
   (see Math 150, Review Topic 3 for help.)

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7. \( \frac{n + 1}{2^n} = \frac{n + 1}{2^{n+1}} \cdot \frac{2^n}{n} = \frac{1}{2} \left( \frac{n + 1}{n} \right) = \frac{n + 1}{2n} \)  
   (see Math 250, Review Topic 1 for help.)

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8. a) 0
b) 0
c) \(-\infty\)
d) \(e\)

(see Math 150, Review Topic 5 for help.)

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9.

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
\theta & 0 & \frac{\pi}{6} & \frac{\pi}{4} & \frac{\pi}{3} & \frac{\pi}{2} & \frac{2\pi}{3} & \frac{3\pi}{4} & \frac{5\pi}{6} & \pi \\
\hline
\sin \theta & 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 1 & \frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{2} & \frac{1}{2} & 0 & \\
\hline
\cos \theta & 1 & \frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{2} & \frac{1}{2} & 0 & \frac{-\sqrt{2}}{2} & \frac{-\sqrt{3}}{2} & -1 & \\
\hline
\tan \theta & 0 & \frac{1}{\sqrt{3}} & 1 & \sqrt{3} & \text{und} & -\sqrt{3} & -1 & \frac{-1}{\sqrt{3}} & 0 \\
\hline
\end{array}
\]

(see Math 150, Review Topics 9, 11, 12 for help.)

Return to Problem
10. \( \sin x \)

\[ f(x) = \sin x \]

\( \cos x \)

\[ f(x) = \cos x \]

\( \tan x \)

\[ f(x) = \tan x \]

(see Math 150, Review Topic 13 for help.)
11.  a) $\sin^{-1}(0) = 0$  
    b) $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$
    
    c) $\arctan(1) = \frac{\pi}{4}$  
    d) $\tan^{-1}(-1) = -\frac{\pi}{4}$
    
    e) $\sin^{-1}(-1) = -\frac{\pi}{2}$  
    f) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

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12. As $x \to \frac{-\pi}{2}$, $\tan x \to \infty$.

    As $x \to \frac{\pi}{2}$, $\tan x \to -\infty$.

    (See the graph of $\tan x$ in the solution to problem 10 above.)

    As $x \to \infty$, $\tan^{-1}(x) \to \frac{\pi}{2}$.

    As $x \to -\infty$, $\tan^{-1}(x) \to -\frac{\pi}{2}$.

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13.  a) $\sin^2 \theta + \cos^2 \theta = 1$  
    b) $\tan^2 \theta + 1 = \sec^2 \theta$
    
    c) $\cos(a + b) = \cos a \cos b - \sin a \sin b$
    
    d) $\sin 2\theta = 2 \sin \theta \cos \theta$
    
    e) $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$
    
    f) $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$

(see Math 150, Review Topics 15 and 16 for help.)
14. a) \( \cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{11\pi}{6} \)

b) \( \tan \theta = -1 \Rightarrow \theta = \frac{3\pi}{4}, \frac{7\pi}{4} \)

(See the table in the solution to problem 9 above.)

c) \( 1 - \sin \theta - 2 \sin^2 \theta = 0 \)

\[
(1 - 2 \sin \theta)(1 + \sin \theta) = 0
\]

\[
\sin \theta = \frac{1}{2} \quad \text{or} \quad \sin \theta = -1
\]

\[
\theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad \text{or} \quad \theta = \frac{3\pi}{2}
\]

(see Math 150,
Review Topic 16
for help.)

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15. If \( \tan \theta = \frac{x}{4} \) where \( x > 0 \), we can draw the following picture.

\[
\begin{align*}
\text{This means the hypotenuse has length } &\sqrt{x^2 + 4}. \text{ Thus,} \\
\sin \theta = &\frac{x}{\sqrt{x^2 + 4}} \text{ and } \cos \theta = \frac{4}{\sqrt{x^2 + 4}}.
\end{align*}
\]

(see Math 150,
Review Topic 9c
for help.)

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