Name: ________________________________
ID number: __________________________
Section number: ______________________
Instructor: _____________________________

Please check to make sure that your copy of the examination has all ten (10) pages and all problems numbered 1 through 15.

SHOW YOUR WORK ON ALL PROBLEMS. INDICATE YOUR ANSWERS CLEARLY.

Only an approved scientific calculator is permitted on this exam. The use of books, notes or other resource materials is not permitted. All cell phones are prohibited during the exam.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Possible</th>
<th>Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>200</td>
<td></td>
</tr>
</tbody>
</table>
1. (10 pts) Evaluate each of the following. Show all work. Simplify your answers to an integer.
   
   (a) $C(9, 4)$
   
   (b) $P(12, 3)$

2. (14 pts) The following data show the number of voters in a sample of 300, categorized by their age and their voter preference.

<table>
<thead>
<tr>
<th></th>
<th>Democrat, D</th>
<th>Republican, R</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young, Y</td>
<td>55</td>
<td>45</td>
<td>100</td>
</tr>
<tr>
<td>Middle-aged, M</td>
<td>80</td>
<td>40</td>
<td>120</td>
</tr>
<tr>
<td>Senior, S</td>
<td>45</td>
<td>35</td>
<td>80</td>
</tr>
<tr>
<td>Totals</td>
<td>180</td>
<td>120</td>
<td>300</td>
</tr>
</tbody>
</table>

(a) Find the probability a person is a Republican.

(b) Find the probability a person is a Senior.

(c) Find the probability a person is a Republican, given that they are a Senior.
In problems 3, 4 and 5, you may leave your answers expressed in terms of products, powers, permutations and/or combinations.

3. (10 pts) Companies whose stocks are listed on the Chicago Stock Exchange have their company name represented by 2 or 3 letters (repetition of letters is allowed). What is the total number of companies that can be listed on the Chicago Stock Exchange?

4. (15 pts) A salesperson contacts prospective customers by telephone, and estimates that 1 in 5 of all telephone calls results in a sale. The salesperson makes 8 telephone calls. Find the probability that
   
   (a) All 8 telephone calls result in a sale.

   (b) None of the 8 telephone calls result in a sale.

   (c) At least 2 of the 8 telephone calls result in a sale.
5. (15 pts) A “hand” of 5 cards is dealt from a regular deck of 52 cards. Find the probability that

(a) The hand contains only black cards.

(b) The hand contains the Jack of Clubs.

(c) The hand contains exactly 2 Jacks.

6. (18 pts) Let $E$, $F$ be events of a sample space $S$ and let $P(\bar{E}) = 0.7$, $P(F) = 0.6$ and $P(E \cap F) = 0.2$.

(a) Fill in all the probabilities in the Venn diagram shown:

(b) Find $P(E \cup F)$

(c) Find $P(\bar{F})$

(d) Find $P(E|F)$

(e) Are $E$ and $F$ independent? Please explain.
7. (18 pts) After a tennis match Jim and Sue leave their bags on the floor. Jim’s bag contains 7 white and 3 green tennis balls. Sue’s bag contains 5 white and 4 green tennis balls. A thief opens a bag at random and steals one tennis ball.

(a) Draw a tree diagram representing this data, listing all outcomes and their probabilities.

(b) What is the probability that the stolen ball is green?

(c) What is the probability that the stolen ball is white?

(d) If the stolen ball is white, what is the probability it was stolen from Sue’s bag?
8. (6 pts) Find the equation of the line passing through $(2, 1)$ and $(-2, 3)$. Write your the answer in slope-intercept form, $y = mx + b$.

9. (10 pts) Find the inverse matrix of

$$
\begin{bmatrix}
1 & 2 & 0 \\
2 & 5 & 0 \\
0 & 0 & 3
\end{bmatrix}
$$
10. (16 pts) Solve the system of linear equations by finding the reduced row-echelon form of the augmented matrix. Label all row operations. Clearly state your final answer.

\[
\begin{align*}
x + y + z &= 1 \\
x + 3y + 3z &= -3 \\
x - y + z &= 3
\end{align*}
\]
11. (10 pts) The following augmented matrices represent systems of linear equations in variables \( x, y \) and \( z \). In each case either state the general solution or that no solution exists.

(a) \[
\begin{bmatrix}
1 & 0 & 0 & | & 1 \\
0 & 1 & 0 & | & 1 \\
0 & 0 & 1 & | & 3
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
1 & 2 & -1 & | & 2 \\
0 & 0 & 1 & | & 7 \\
0 & 0 & 0 & | & 0
\end{bmatrix}
\]

12. (14 pts) Formulate as an LP model. DO NOT ATTEMPT TO SOLVE IT.

A toy manufacturer makes two different kinds of model cars: the Honda and the Ford. They are made of steel and aluminum. Each Honda requires 2 units of steel and 1 unit of aluminum, each Ford requires 3 units of steel and 4.5 units of aluminum. The company has 7,000 units of steel and 11,000 units of aluminum available. The number of Hondas made must be at most the number of Ford made. The total number of model cars made must be between 1,000 and 2,000. If $3 profit is made on each Honda and $5 on each Ford, how many of each model car should the toy manufacturer make in order to maximize its profit?
13. (14 pts) Consider the following linear programming problem

Maximize $P = 4x + y$

subject to the constraints

$3x - y \geq 6$
$x + y \leq 6$
$x \geq 0, y \geq 0$

(a) Sketch the feasible region labeling all corner points.

(b) Solve the problem **geometrically** and report the complete solution.
14. (10 pts) For each of the following tableaux, perform one of the following steps:
1) If a pivot is required, circle the pivot element and write ‘pivot needed’ but DO NOT PIVOT.
2) If there is no optimal solution, say so and say why.
3) If the problem is finished, report the complete solution.

(a)

\[
\begin{array}{cccccccc}
\text{BV} & P & x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & \text{RHS} \\
\hline
s_1 & 0 & 3 & 0 & 4 & 1 & -2 & 0 & 2 \\
x_2 & 0 & -1 & 1 & 0 & 0 & 1 & 0 & 4 \\
s_3 & 0 & 4 & 0 & -1 & 0 & -1 & 1 & 1 \\
P & 1 & -5 & 0 & -1 & 0 & 3 & 0 & 7 \\
\end{array}
\]

(b)

\[
\begin{array}{cccccccc}
\text{BV} & P & x_1 & x_2 & s_1 & s_2 & \text{RHS} \\
\hline
x_1 & 0 & 1 & -2 & 1 & 0 & 4 \\
s_2 & 0 & 0 & -5 & 3 & 1 & 18 \\
P & 1 & 0 & -8 & 3 & 0 & 12 \\
\end{array}
\]
15. (20 pts) Use the **simplex method** to solve the following LP model:

Maximize $P = 5x_1 + 4x_2 + 3x_3$

subject to the constraints

\[5x_1 + 3x_2 + 5x_3 \leq 20\]
\[x_1 + x_2 + 5x_3 \leq 6\]
\[x_1 \geq 0, x_2 \geq 0, x_3 \geq 0\]