We prove well-posedness of time-dependent Ginzburg–Landau superconductivity equations in a nonconvex polygonal domain, and decompose the solution as a regular part plus a singular part. We see that the magnetic potential is not in $H^1$ in general, and the standard finite element method may give incorrect solutions. To remedy this situation, we reformulate the equations into an equivalent system based on the Hodge decomposition, which avoids direct calculation of the singular magnetic potential. The essential unknowns of the reformulated system admit $H^1$ solutions and can be solved correctly by the standard finite element methods. We then propose a decoupled and linearized FEM to solve the reformulated equations and prove the convergence of the numerical solution based on proved regularity of the solution.