Quantum entanglement is known to provide advantage in many nonlocal scenarios. However, in practice, finding the best entangled strategy is challenging and one is often forced to resort to ad hoc methods. In general, the mathematical structure of the set of entangled strategies is poorly understood and many basic questions remain open.

One basic open question is whether a continuous payoff function should always achieve its maximum when optimized over the set of entangled strategies. A positive answer would alleviate the search of optimal entangled strategies while a negative one would give evidence of the hardness of this problem. In this work, we show that the answer can be negative even if the nonlocal task has classical inputs and outputs. In particular, we present a one-round two-party nonlocal game at which entangled quantum parties can perform increasingly better by sharing a quantum system of increasingly larger size. Although no error-free strategy exists for this game, the players can succeed with probability arbitrarily close to one by using entangled states of increasingly larger dimension.

This is joint work with Thomas Vidick.