This talk will present a formulation and a solution of a regular generalized Sturm-Liouville problem using the variational method. The problem is formulated in terms of generalized operators introduced recently. Although, many different kernels can be considered, the kernels considered include fractional power kernel which lead to fractional integral and derivative operators; fractional power multiplied with exponential kernel which leads to generalized fractional integral and derivative operators with weights and scales, and some arbitrary kernels. In special cases, these operators reduce to Riemann-Liouville fractional integral and Riemann-Liouville and Caputo differential operators. The fractional power and the fractional power with exponential function kernels are singular kernels, and they lead to operators which satisfy semi-group properties. However, in general the generalized operators which are analogous to the fractional integral operators do not satisfy the semi-group property. Some properties of these operators are discussed.

Next, it will be demonstrated that the regular generalized Sturm-Liouville problem has infinite countable set of positive eigenvalues, and the eigenfunctions associated with distinct eigenvalues form a set of orthonormal bases for square integral functions. Thus, the regular generalized Sturm-Liouville problem considered here exhibit properties similar to Sturm-Liouville problems defined using integer order derivatives.