

Math 101 Final Exam Study Notes:

Please remember there is a large set of final exam review problems in Doc Sharing (under Course Tools in MLP). Highlighted are what might be considered formulas

I. Graph Theory

A. Formulas

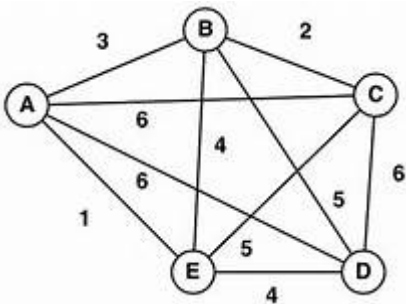
- 1) A tree with n vertices has $n-1$ edges
- 2) A complete graph with n vertices has $(n-1)!$ Hamiltonian Circuits. A complete graph is one in which each vertex shares an edge with every other vertex.

B. Concepts

- 1) In order for a graph to have an **Euler circuit**, it must have all even vertices. An Euler circuit traverses each edge exactly once, starting and ending at the same vertex.
- 2) In order for a graph to have an **Euler path**, it must have exactly two odd vertices. An Euler path traverses each edge exactly once, starting and ending at different vertices.
- 3) The **degree** of a vertex = number of edges coming out of vertex.
- 4) **Adjacent vertices** share an edge.
- 5) **Adjacent edges** share a vertex.

C. Algorithms

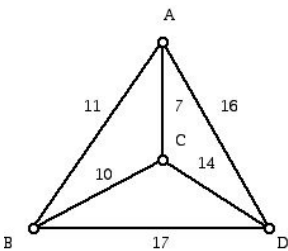
- 1) Use **Kruskal's Algorithm** to find the minimal spanning tree. Keep grabbing the cheapest edge (start anywhere) until you have a connected graph with no circuits. You CAN have more than 2 edges coming out of a vertex.
- 2) Three methods for finding the least cost Hamiltonian Circuit. The only one that guarantees the least cost circuit is the Brute Force Method.
 - A) **Nearest Neighbor** – start at the vertex indicated. Go to closest neighbor (least weight) and continue until you make a circuit. Cannot have more than 2 edges coming out of a vertex and don't close circuit too early. There should be only ONE circuit.
 - B) **Best Edge Algorithm (cheapest link)** – Don't start at a particular vertex. Highlight least cost edge. Continue doing this (don't need to stay connected during process). You are done when you have only one circuit. Cannot have more than 2 edges coming out of a vertex and don't close too early.
 - A) **Brute Force** – list all possible Hamiltonian circuits. Add up weights to find cheapest one.



Example 1: Find approximate minimal cost circuit using

- A) Nearest neighbor starting at A
- B) Best edge, write answer starting with A

Example 2: Find least cost Hamiltonian Circuit using Brute Force Method.



II. *Problem Solving/fractions*

A) Concepts:

1) **Convert between decimal and percent**

- a) Percent to decimal – divide by 100
- b) Decimal to percent – multiply by 100

2) **Convert between fractions and decimals/percents**

- a) Fraction to decimal, divide numerator by denominator.
- b) Fraction to percent, divide numerator by denominator then multiply by 100.

Example 3: Convert

a) $4/5$ to a percent

b) 8 to a percent

- 3) To find the percent of something, change the percent to a decimal (divide by 100) and multiply by that something.
- 4) Understanding that **profit** is the difference between how much was made and how much was spent.
- 5) Understand that **linear** means that something is increasing by a fixed amount or decreasing by a fixed amount (straight line)

III. *Probability*

A) Concepts:

- 1) Fundamental counting principal. If you can choose one item from a group of M items, one from a group of N items, one from a group of P items, then the total number of three item choices is $M \times N \times P$.

Example 4: If a code consists of 3 digits and one of five symbols, how many possible codes are there?

2) Probability = (how many in the event)/(total number of items).

A) If two events are happening, remember **AND means multiply and OR means add.**

If you want the probability the first even happens and the 2nd event happens, you must pay attention if the first event effects the probability of the 2nd event (with or without replacement).

Example 5: Two balls are chosen from 5 red and 8 blue and 2 yellow. Find the probability:

- First is red and 2nd is blue if first is replaced before 2nd is drawn. First is red and 2nd is blue if first is not replaced before 2nd is drawn.
- What is the probability a randomly selected ball is red or yellow?

3) **Writing out the sample space** and use it to find a probability.

Example 6:

- A couple has 3 children, all possible combinations of boys and girls. Then find P(all girls)
- Answers on a true false test if there are two questions on the test. Then find P(both false)
- Rolling a die and flipping a coin. Then find P(head and rolling a 1)

4) **Probability in genetics.** Draw out a Punnet Square and use it to find the probability.

Example 6: Suppose R= red gene, r = white gene. Red is dominant. If Rr is crossed with rr, what is probability offspring is white?

5) **“Or”** for events that are not mutually exclusive. Make sure to not double count events.

Example 7: Find the probability of selecting a king or a diamond randomly from a deck of cards

6) **Odds.** Remember that odds are # in favor of event : # not in favor of event. This is different than probability. Probability is # in favor of event/total number of possibilities.

Example 8: If there are 3 equally sized red sections, 6 green sections and 3 yellow sections on a spinner:

What is the probability of landing on red?

What are the odds of landing on red?

What are the odds of not landing on red?

- 7) **Conditional Probability.** Remember that the denominator changes with conditional probability. The denominator total is the number in the “given” event.

Example 9: Use the table to answer the following.

	Spanish	French	German	Total
Boys	10	2	8	20
Girls	15	12	3	30
Total	25	14	11	50

- Find the probability that a randomly chosen student is a boy.
- Find the probability that a randomly chosen student is studying Spanish?
- Find the probability that a randomly chosen student is a boy given the student is studying Spanish?
- Find the probability that a randomly chosen student is not studying Spanish?

- 8) **Expected Value.** Know that the expected value is: Sum of the probability times the outcome with that probability.

Example 10: Tomatoes can be sold in packs of various weights of kg. The probability of a distributor purchasing the different weights are given. Find the expected number of kg purchased.

Demand	Probability
0 kg	.05
25 kg	.10
50 kg	.30
75 kg	.35
100 kg	.20

Example 11: If Bob wins \$3 if a 6 is rolled and loses his \$1 he paid to play if anything else is rolled, what is Bob's expected net winnings?

IV. Voting

1) Methods of voting:

Plurality = most first place votes wins

Majority = must have more than half of votes

Plurality with elimination = eliminate candidate with least number of first place votes. Draw a new table. Eliminate candidate with least number of first place votes. Draw a new table. Do this until only one winner left.

Borda Count = assign last place votes = 1 point, 2nd to last = 2 point, etc. Count up points for each candidate. Largest number of points wins.

Pairwise comparison (head to head) = **Remember that there are $n(n-1)/2$ possible comparisons.** Compare each candidate with every other. The one that wins the most comparison wins the overall vote.

Know the definitions:

Monotonicity – If a candidate wins and in a reelection, the only changes favor that candidate, that the candidate should win reelection.

Irrelevant Alternatives – If a candidate wins and in a recount, the only changes are that one or more of the other candidates are removed from the ballot, then the candidate should still win the election.

Example 12:

	35	28	20	17	number of voters
1st	N	S	J	J	
2nd	S	N	N	S	
3rd	J	J	S	N	

Who wins vote by:

- a) Plurality?
- b) Plurality with elimination?

d) Pairwise Comparison?

c) Borda count?

V. Statistics

1) Definitions/formulas

Population - all people or objects whose properties are to be described.

Sample - subset of the population.

Statistics - describe samples (Ex. 35% of the 400 people polled like brand A toothpaste)

Parameters - describe populations (ex. 35% of all people like Brand A toothpaste)

Mean = add up data values and divide by total number of data values (average)

Median = middle data value

Mode = most frequently appearing data value (no mode if none appear more than once, can be multiple modes if more than one appears the most frequent)

Outliers – data values that are much higher or lower than rest of data values.

Midrange: $(\text{highest} + \text{lowest})/2$

Range: highest – lowest

Standard deviation: 1) find the mean of the n data values 2) take each data value from the mean and square these. 3) add up these results 4) divide this sum by (n-1) 5) take square root.

The standard deviation is on average how far the data values are from the mean.

Five number summary: Min, Q1, Med, Q3, Max

Find min, max, and median first. To find Q1, find median of first half of numbers (cover up true median). To find Q3, find the median of the 2nd half of the numbers (cover up true median)

Example 14: Given data set:

1, 4, 6, 8, 10, 10, 14, 15, 18, 20, 80

Find:

- a) outlier(s)
- b) range
- c) midrange
- d) 5 number summary

Example 15: Given 4, 6, 1, 9 find standard deviation

to 2 decimal places.

2) Stem-and-leaf plot

Be able to read them.

The ages of males and females living in a neighborhood are given.

Example 13

Male		Female
5, 2, 0	1	5, 8
5, 1	2	1, 6, 9, 9
5, 5, 5, 3, 1	3	
5, 2	4	1, 2, 6, 8
9, 8, 6, 1, 1	5	5
6, 5, 5, 0	6	0, 1
2, 1, 1, 0, 0	7	2

a) What is the age of the oldest male?

b) Find the median female age

c) Find the mode for the male ages

3) Calculating GPA

Multiply each numeric grade (4=A, 3=B, 2=C, 1=D, 0=F) by the number of credits the course was worth. Add these together. Divide this result by number of credit hours taken. Round to two decimal places.

Example 14: Find the GPA on 4.0 scale : A in science (3 credit hours), B in history (3 credit hours), C in math (4 credit hours), A in health (2 credit hours).

- 4) Solve **normal distribution** problems. Always draw bell curve, label mean, and shade region you want to find (the percent).
- If data value(s) are 1, 2, or 3 standard deviations away from the mean, use the 68-95-99.7% rule to find the percents asked for.
 - If data value(s) are not 1,2, or 3 standard deviations away from the mean, then use $z = (\text{data value} - \text{mean}) / \text{standard deviation}$. Look up the z-score(s) in the table provided.

If you have shaded area to the left (lower than the score), then that is your answer.

If you have shaded area to the right (higher than the score), then subtract the percent from chart from 100%.

If you have shaded the area between two scores, then subtract the two percents you found in the table.

Example 15: Suppose that the distribution of diastolic blood pressure in a population of hypertensive women is modeled well by a normal probability distribution with mean 100 mm Hg and standard deviation 14 mm Hg.

- a) Use 68-95-99.7 rule to find percent of women with diastolic blood pressure between 72 and 128.
- b) What percent of women have diastolic blood pressure between 86 and 100 mm Hg?
- c) What percent of women have diastolic blood pressure more than 110 mmHg?

VI. Financial Math

Know:

- 1) To find the percent of a number, change the percent to a decimal and multiply by the number.
- 2) To find the **percent increase or decrease = (difference between old and new amount)/(old amount)** change this to a percent by multiplying by 100%.

Example 16: If Bob weighed 182 pounds and after 3 months weighed 173 pounds. What is the percent decrease in weight?

Vocabulary:

Gross income = all money made

Adjusted gross income = gross income minus adjustments (adjustments are non-taxed items).

Taxable income = adjustment gross income - exemptions - deductions * (remember to choose larger of standard deduction or itemized deduction; exemption is a fixed number).

Income tax is found by using the tax chart and your taxable income. Subtract off any tax credits at the end.

Net income = adjusted gross income minus all money paid in taxes.

For investment/loan problems:

- 1) Decide if it is a simple interest problem. If so, use formulas 1 or 2.
- 2) If it is a compound interest problem with n being a finite number and you are
 - a) Depositing money one time, use formula 3, no matter what you are trying to solve for.
 - b) Depositing money multiple times, use formula 5 (if solving for A) or 6 if solving for P .
 - c) Borrowing money one time, then use formula 3, no matter what you are trying to solve for.
 - d) Borrowing money to pay back in MULTIPLE payments, use loan formula #7.
- 3) If it is a compound interest problem with continuous compounding, then use formula 4.

There is no formula for interest in compound interest problems. Figure out the difference between how much you put in and how much you have at the end or how much you borrowed and how much you paid back!

Know how to fill in loan table (use $I=Prt$ where $t = 1/12$ because the table is in month format)

Example 17: Assume you borrow \$10,000 from a lender for an automobile purchase. The lender offers you an Annual Interest Rate (AIR) of 12% and you agree to pay \$350 each month. Fill in first 2 months of loan amortization table.