MATH 519 QUALIFYING EXAM

FALL 2010

Directions. Answer 8 of the following 10 questions. Begin each question on a fresh sheet of paper. Hand in only the 8 questions you wish to have graded.

1. Suppose the group G has a subgroup H of order 5 and a normal subgroup N of order 7. Prove G has an element of order 35.

2. Let S_n be the group of permutations on $\{1, 2, \ldots, n\}$. Let S_{n-1} be the subgroup of S_n consisting of permutations on $\{1, 2, \ldots, n-1\}$. If H is a subgroup of S_n with $S_{n-1} \subset H \subset S_n$ then prove that either $S_{n-1} = H$ or $H = S_n$.

3. Prove no group of order $132 = 2^2 \cdot 3 \cdot 11$ is simple.

4. Prove that every p-group has a nontrivial center.

5. Classify all abelian groups of order 180 and give at least three nonabelian groups of order 180. Prove that the nonabelian groups are nonisomorphic.

6. Suppose that G is a nonabelian group, and $|G| < 100$ is divisible by only two primes. If one of the primes is 17, prove that the other one is 2.

7. Let R be a commutative ring with unity. Let $I \subset R[x]$ be an ideal. Let n be the least degree of a non-zero element of I and suppose I contains a monic polynomial of degree n. Prove I is principal.

8. Let D consist of the polynomials in $\mathbb{Z}[x]$ with the coefficient of x equal to 0. You may assume that D is an integral domain. Prove D is not a UFD.

9. An element x of a ring R is said to be *nilpotent* if $x^n = 0$ for some positive integer n. Prove that the set of nilpotent elements of a **commutative** ring is an ideal.

10. Let R_1, R_2 be two commutative rings with 1, and let $R = R_1 \times R_2$. Prove that every prime ideal of R contains either $R_1 \times \{0\}$ or $\{0\} \times R_2$.