Ph. D. Qualifying Examination in Statistics
Tuesday, August 26, 2003

1) Suppose that $X_1, ..., X_n$ are iid normal distribution with mean 0 and variance σ^2. Consider the following estimators: $T_1 = \frac{1}{2}|X_1 - X_2|$ and $T_2 = \sqrt{\frac{1}{n}\sum_{i=1}^n X_i^2}$.

a) Is T_1 unbiased for σ? Evaluate the mean square error (MSE) of T_1.
b) Is T_2 unbiased for σ? If not, find a suitable multiple of T_2 which is unbiased for σ.

2) Let $X_1, ..., X_n$ be independent identically distributed random variables with pdf (probability density function)

$$f(x) = \frac{1}{\lambda}\exp\left(-\frac{x}{\lambda}\right)$$

where x and λ are both positive. Find the uniformly minimum variance unbiased estimator (UMVUE) of λ^2.

3) Let $X_1, ..., X_n$ be a random sample from a population with pdf

$$f(x) = \begin{cases} \frac{\theta x^{\theta-1}}{3^\theta} & 0 < x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

The method of moments estimator for θ is $T_n = \dfrac{\overline{X}}{3 - \overline{X}}$.

a) Find the limiting distribution of $\sqrt{n}(T_n - \theta)$ as $n \to \infty$.

b) Is T_n (asymptotically) efficient? Why?

c) Find a consistent estimator for θ and show that it is consistent.

4) Let $X_1, ..., X_n$ be independent identically distributed random variables with pdf

$$f(x) = \frac{1}{\lambda}\exp\left[-(1 + \frac{1}{\lambda})\log(x)\right]$$

where $\lambda > 0$ and $x \geq 1$.

a) Find the maximum likelihood estimator of λ.

b) What is the maximum likelihood estimator of λ^8 ? Explain.

5) Let $X_1, ..., X_n$ be independent identically distributed random variables from a distribution with pdf

$$f(x) = \frac{x^2 \exp\left(\frac{-x^2}{2\sigma^2}\right)}{\sigma^3 \sqrt{2}\ \Gamma(3/2)}$$

where $\sigma > 0$ and $x \geq 0$.

a) What is the UMP (uniformly most powerful) level α test for $H_o : \sigma = 1$ vs. $H_1 : \sigma = 2$?

b) If possible, find the UMP level α test for $H_o : \sigma = 1$ vs. $H_1 : \sigma > 1$.

6) Suppose that $X_1, ..., X_n$ are iid with the Weibull distribution, that is the common pdf is

$$f(x) = \begin{cases} \frac{b}{a} x^{b-1} e^{-\frac{x^b}{a}} & 0 < x \\ 0 & \text{elsewhere} \end{cases}$$

where a is the unknown parameter, but $b (> 0)$ is assumed known.

a) Find a minimal sufficient statistic for a

b) Assume $n = 10$. Use the Chi-Square Table and the minimal sufficient statistic to find a 95% two sided confidence interval for a.