

1. (8 pts) Evaluate each of the following. Show all work. Simplify your answers to an integer.

(a) $7!$

(b) $C(10, 4)$

2. (16 pts) A survey of personal computer users gave the following results:

27 use IBM,

35 use Apple,

35 use Dell,

10 use both IBM and Apple,

8 use both IBM and Dell,

12 use both Apple and Dell,

4 use all three,

and 29 use another brand.

(a) Represent this information on the Venn Diagram shown:

(b) How many users use only IBM or only Apple or only Dell?

(c) How many users use IBM and Apple but not Dell?

(d) How many users were surveyed in total?

In problems 3, 4 and 5, you may leave your answers expressed in terms of products, powers, permutations and/or combinations. You need not simplify your answers.

3. (10 pts) A palindrome is a 'word' (real or imaginary) that reads the same forwards as backwards. *eg*, CIVIC, BCXCB, AABAA, AAAAA are 5-letter palindromes.

(a) How many 5-letter palindromes are there?

(b) How many 5-letter palindromes end in Z?

4. (15 pts) When Mr.Fishhook goes on a fishing trip, his probability of catching a fish is 85%. If he makes 20 fishing trips, find the probability that

(a) He catches fish on exactly 12 trips.

(b) He catches fish on at least 2 trips.

(c) He catches fish on no trips.

5. (15 pts) A “hand” of 5 cards is dealt from a regular deck of 52 cards. Find the probability that

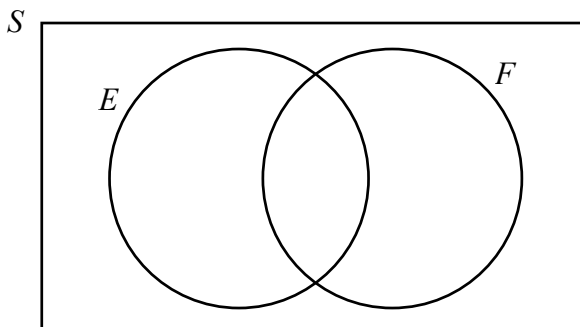
(a) The hand contains only Diamonds.

(b) The hand contains the Ace of Diamonds.

(c) The hand is a ‘full-house’ (two of a kind, and three of a kind).

6. (18 pts) Let E, F be events of a sample space S and let $P(E) = 0.3$, $P(\bar{F}) = 0.6$ and $P(E \cap F) = 0.1$.

(a) Fill in all the probabilities in the Venn diagram shown:



(b) Find $P(E \cup F)$

(c) Find $P(\bar{E} \cap F)$

(d) Find $P(E|F)$

(e) Are E and F independent? Please explain.

7. (18 pts) If a person with tuberculosis (TB) is given a TB-screening, the probability that his or her condition will be detected is .8. If a person without TB is given a TB-screening, the probability that he or she will be diagnosed incorrectly as having TB is .1. Suppose further that 5% of the adults in a certain city have TB.

(a) Draw a tree diagram representing this data, listing all outcomes and their probabilities.

(b) What is the probability that a randomly chosen person in the city is diagnosed as having TB?

(c) If an adult is diagnosed as having TB, what is the probability that he or she actually has TB?

8. (10 pts) (a) Find the equation of the line passing through $(3, -2)$ and $(2, 1)$. Write the answer in slope-intercept form, $y = mx + b$.

(b) Determine whether the two lines $x + 4y = 3$ and $2x + 5y = 7$ are intersecting, parallel, or coincident.

9. (12 pts) Given the two matrices $A = \begin{bmatrix} 1 & 1 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & \frac{1}{2} \\ 1 & -1 & 4 \end{bmatrix}$.

(a) Find the inverse matrix A^{-1}

(b) Check your inverse from (a).

(c) Find AB (if possible)

(d) Find BA (if possible)

10. (14 pts) Solve the system of linear equations by finding the reduced row-echelon form of the augmented matrix. Label all row operations. Clearly state your final answer.

$$\begin{array}{rclcl} x & & + & z & = & 1 \\ & & & y & + & 2z & = & 7 \\ 2x & + & y & + & 3z & = & 7 \end{array}$$

11. (12 pts) The following augmented matrices represent systems of linear equations in variables x , y and z . In each case either state the general solution or that no solution exists.

$$(a) \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -6 \end{array} \right]$$

$$(b) \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & -4 \end{array} \right]$$

$$(c) \left[\begin{array}{ccc|c} 1 & 4 & 0 & 1 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

12. (14 pts) Formulate as an LP model. DO NOT ATTEMPT TO SOLVE IT.

A toy company makes two different kinds of face masks: Dracula mask and Wolfman mask, using hair, plastic, and latex. A Dracula mask requires 4 oz hair, 15 oz of plastic, and 5 oz of latex. A Wolfman mask requires 3 oz hair, 18 oz of plastic, and 6 oz of latex. The company has 10,000 oz hair, 20,000 oz of plastic, and 15,000 oz of latex available. At least as many Dracula masks as Wolfman masks must be made, and, because of heavy customer demand, the total number of masks made must be at least 20,000. If \$6 profit is made on each Dracula mask and \$5 on each Wolfman mask, how many of each mask should the toy company make in order to maximize its profit?

13. (12 pts) Consider the following linear programming problem

$$\text{Maximize } P = -3x + 2y$$

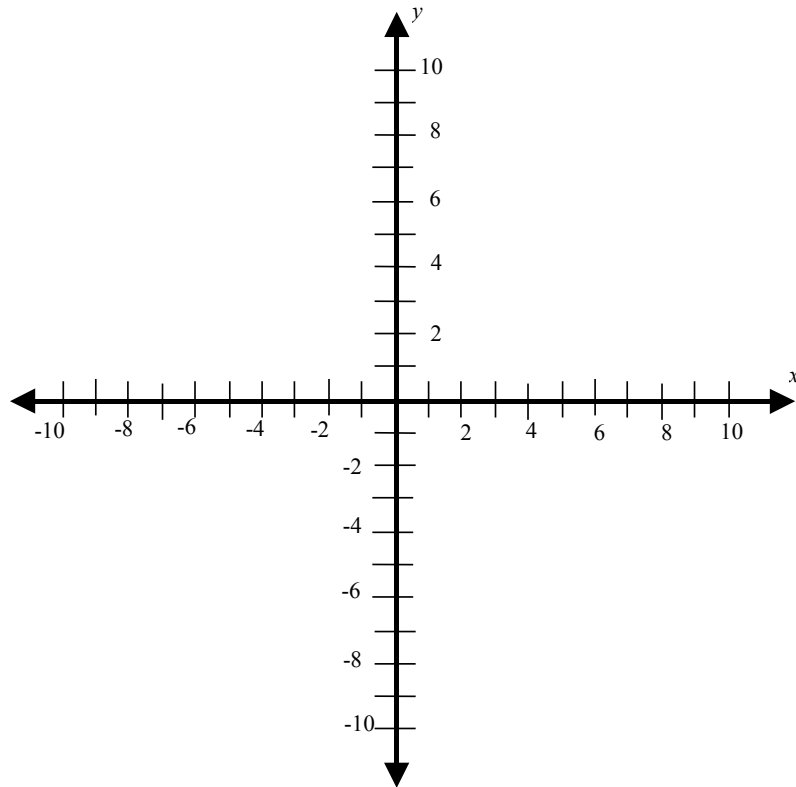
subject to the constraints

$$3x + y \leq 10$$

$$2x - y \leq 3$$

$$x \geq 0, y \geq 0$$

(a) Sketch the feasible region labeling all corner points.



(b) Solve the problem **geometrically** and report the complete solution.

14. (8 pts) For each of the following tableaux, perform one of the following steps:

- 1) If a pivot is required, circle the pivot element and write 'pivot needed' but DO NOT PIVOT.
- 2) If there is no optimal solution, say so and say why.
- 3) If the problem is finished, report the complete solution.

(a)

$$\begin{array}{c} \text{BV} \\ x_1 \\ s_2 \\ P \end{array} \left[\begin{array}{cccccc|c} P & x_1 & x_2 & s_1 & s_2 & \text{RHS} \\ 0 & 1 & 6 & 1 & 0 & 7 \\ 0 & 0 & 5 & -1 & 1 & 2 \\ \hline 1 & 0 & 3 & 4 & 0 & 12 \end{array} \right]$$

(b)

$$\begin{array}{c} \text{BV} \\ s_1 \\ x_2 \\ s_3 \\ P \end{array} \left[\begin{array}{cccccc|c} P & x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & \text{RHS} \\ 0 & 3 & 0 & 4 & 1 & -2 & 0 & 2 \\ 0 & -1 & 1 & 0 & 0 & 1 & 0 & 4 \\ 0 & 4 & 0 & -1 & 0 & -1 & 1 & 2 \\ \hline 1 & -4 & 0 & -1 & 0 & 3 & 0 & 8 \end{array} \right]$$

15. (18 pts) Use the **simplex method** to solve the following LP model:

$$\text{Maximize } P = 3x_1 + 2x_2 + 2x_3$$

subject to the constraints

$$2x_1 + x_2 + 4x_3 \leq 4$$

$$x_1 - x_2 + 6x_3 \leq 8$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$