NO CALCULATORS OR CELL PHONES ALLOWED.

**Write a coherent, well organized, properly notated process or you <u>will not</u> receive credit for your answer.
ALL work must be on this exam to be graded.
If you use scratch paper, you MUST transfer your work to this test.**

**Part I. Answer all problems in this section. Little partial credit will be given.
[6 points each]**

1. Write the following inequalities in interval notation:

a. $x \geq 0$

$[0, \infty)$

b. $-2 < x \leq 5$

$(-2, 5]$

c. $x \neq 10$

$(-\infty, 10) \cup (10, \infty)$

2. Let $f(x) = 3x - 2$ and $g(x) = x^2 + 2$. Find and simplify

a. $(f - g)(1)$

$= f(1) - g(1)$

$= [3(1) - 2] - [(1)^2 + 2]$

$= 3 - 2 - 1 - 2$

$= -2$

b. $(f \circ g)(x)$

$= f(g(x))$

$= f(x^2 + 2) = 3(x^2 + 2) - 2$

$= 3x^2 + 6 - 2$

$= 3x^2 + 4$

3. Simplify the following to $a + bi$ form.

a. $(1 + 2i)(3 - i)$

$= 3 + 6i - i - 2i^2$

$= 3 + 5i + 2$

$= 5 + 5i$

b. $(2i)^3$

$= 8i^3 = -8i$

4. Evaluate:

a. $\ln e^{2x-1} = 2x - 1$

b. $\log 1 = 0$

c. $\log_3 9 = 2$

5. Given the piecewise function:

$$f(x) = \begin{cases} 3x - 1, & for\ x > 2 \\ -1, & for\ x \leq 2 \end{cases}$$

Fill in the table:

x	$f(x)$
1	-1
2	-1
3	$3(3) - 1 = 8$

6. Write an equation for a function that has a graph with the given characteristics:

a. The shape of $y = x^2$ but upside-down and shifted right 7 units.

$$y = -(x - 7)^2$$

b. The shape of $y = \sqrt{x}$ but reflected across the y-axis.

$$y = \sqrt{-x}$$

7. Solve for x:

$|3x + 4| - 2 = 10$

$$|3x + 4| = 12$$

$$3x + 4 = 12 \quad and \quad 3x + 4 = -12$$

$$x = \frac{8}{3} \quad and \quad x = -\frac{16}{3}$$

8. Solve for x:

$x^2 + 25 = 0$

$x = \pm 5i$

9. Thara's T-shirt store sold 36 shirts one day. All blue T-shirts cost \$12 each and all green shirts cost \$18 each. Total receipts for the day were \$522. How many of each color of shirt were sold?

   Set up a system – but you do not need to solve!

   $b + g = 36$

   $12b + 18g = 522$

10. Find a 3rd degree polynomial (in polynomial form) with zeros: 2 and $-5i$.
    Write the polynomial on the line below.

    $-5i$ is a zero implies that $5i$ is also a zero

$$f(x) = (x - 2)(x + 5i)(x - 5i)$$

$$(x - 2)(x^2 + 25)$$

$$f(x) = x^3 - 2x + 25x - 50$$

Part II. Answer all problems in this section. [10 points each]
Some partial credit will be given based on work shown.

11. State the slope of the following lines:

a. $x + 2y = 9$

$2y = -x + 9$

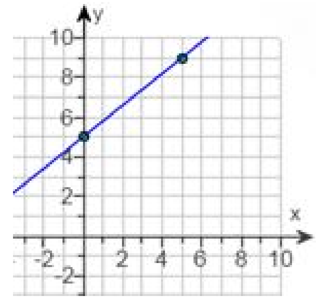
$y = -\frac{1}{2}x + \frac{9}{2}$

$m = -\frac{1}{2}$

b.

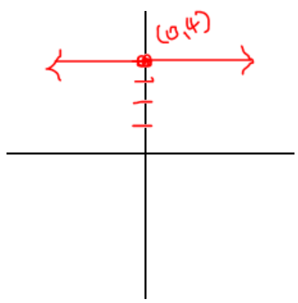
(0,5) & (5,9)

$m = \frac{9-5}{5-0} = \frac{4}{5}$



12. Graph the following lines:

a. $y = 4$



b. $y = -2x - 1$

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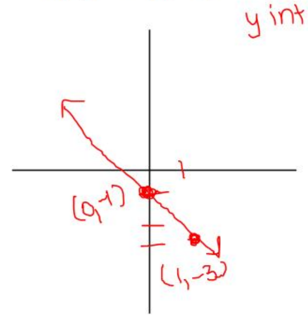


y intercept

(0,-1)

x = 1

→ y = -2-1 = -3

(1,-3)

13. Given $f(x) = x^2 - 2x - 3$, find the following and sketch the graph. Be sure to label all intercepts and the vertex.

Vertex: (__1__, __-4__)

$\frac{-(-2)}{2(1)} = 1$ $f(1) = 1-2-3$

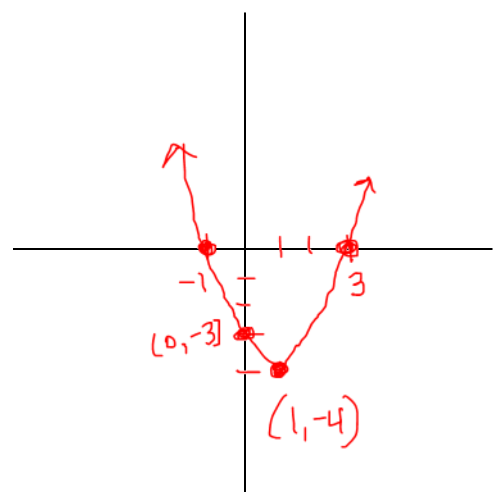
$= -4$

Zeros: -1, 3

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

y-intercept: (0, -3)



14. Given a function $f(x)$, find the inverse function $f^{-1}(x)$. Show all work algebraically and circle your answers.

a. $f(x) = \sqrt[3]{2x+3}$

$x = \sqrt[3]{2y+3}$

$x^3 = 2y + 3$

$\frac{x^3 - 3}{2} = y$

$f^{-1}(x) = \frac{x^3 - 3}{2}$

b. $f(x) = 5x^3 - 8$

$x = 5y^3 - 8$

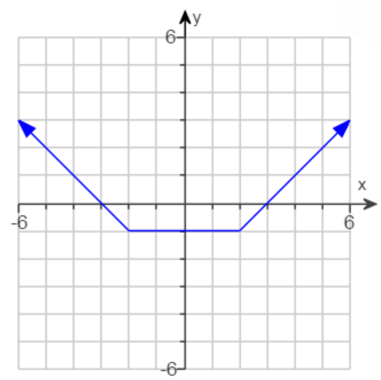
$\frac{x+8}{5} = y^3$

$\sqrt[3]{\frac{x+8}{5}} = y$

$f^{-1}(x) = \sqrt[3]{\frac{x+8}{5}}$

15. Give the following intervals, in interval notation, based on the function $f(x)$ shown in the graph.

a. Domain of f: $(-\infty, \infty)$	b. Range of f: $[-1, \infty)$
c. f is increasing: $(2, \infty)$	d. f is constant: $(-2,2)$
e. $f(x) \leq 0$ $[-3,3]$	



16. Solve for x.

a. $8^{x-1} = \frac{1}{2}$

$(2^3)^{x-1} = 2^{-1}$

$3(x-1) = -1$

$3x - 3 = -1$

$x = \frac{2}{3}$

b. $2e^{x+5} + 3 = 9$

$e^{x+5} = \frac{9-3}{2} = 3$

$\ln e^{x+5} = \ln 3$

$x + 5 = \ln 3$

$x = \ln 3 - 5$

17. Find the following and sketch the graph of $f(x) = \frac{-x}{x-4}$. Be sure to label all asymptotes and intercepts and use additional points as needed.

$f(5) = \frac{-5}{5-4} = -5$

Zeros:

$-x = 0$

$\rightarrow x = 0$

y-int:

$f(0) = \frac{0}{-4} = 0$

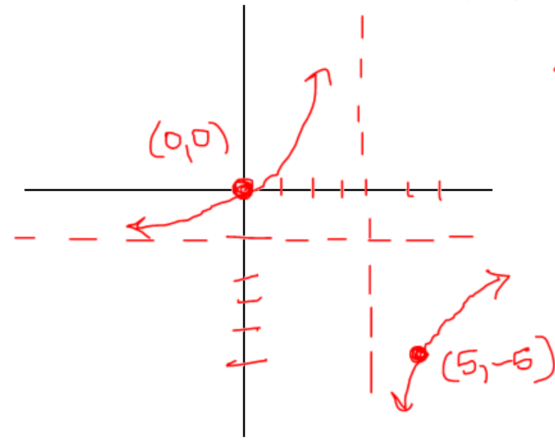
HA:

$\frac{-x}{x} = -1$

$y = -1$

VA:

$x = 4$



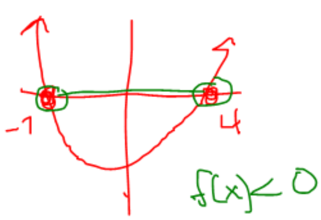
18. Solve the following inequalities. State your answers in interval notation.

a. $x^2 + 3x - 28 < 0$

$(x+7)(x-4) = 0$

$x = -7, 4$

Graph



$-7 < x < 4$

$(-7, 4)$

b. $-3 \leq \frac{x+5}{2} < 14$

$-6 \leq x + 5 < 28$

$-11 \leq x < 23$

$\boxed{[-11, 23)}$

19. Solve the following system. Write your answer as an ordered pair.

$$\begin{cases} 5x + y = -20 \\ 7x - 3y = -50 \end{cases}$$

$eq1 * 3 \rightarrow \quad 15x + 3y = -60$

$\underline{eq2 \rightarrow \quad 7x - 3y = -50}$

$22x = -110$

$x = -\frac{110}{22} = -5$

$5(-5) + y = -20 \rightarrow \quad y = 5$

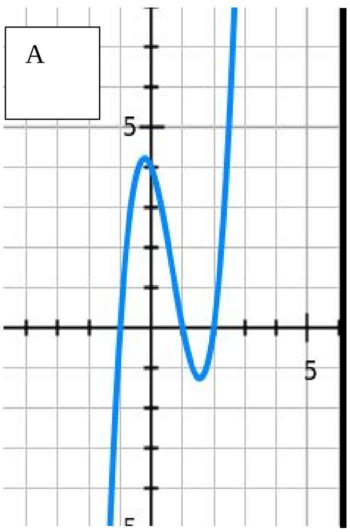
Solution: $(-5, 5)$

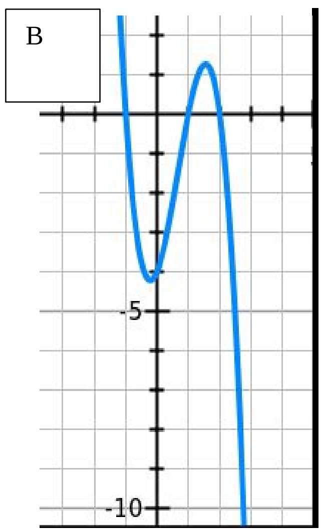
20. Match the equation with the graph below. Put letter choice in answer blank.

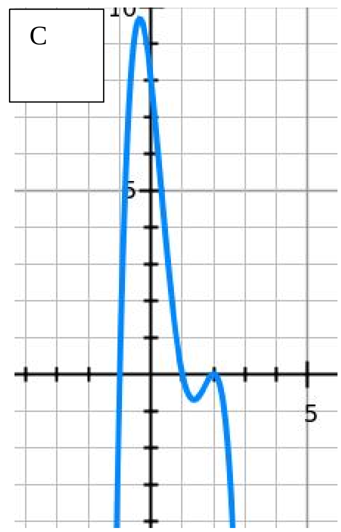
$y = 2(x-2)(x+1)(x-1)$ Answer: A

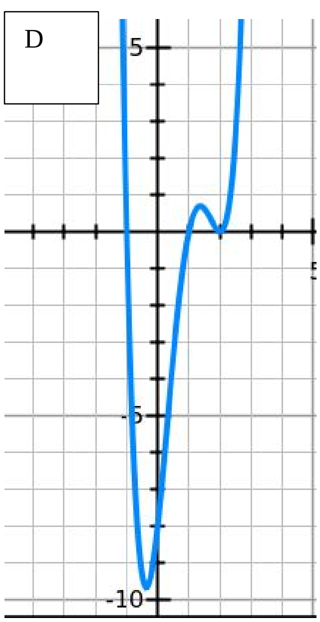
$y = -2(x-2)^2(x+1)(x-1)$ Answer: C

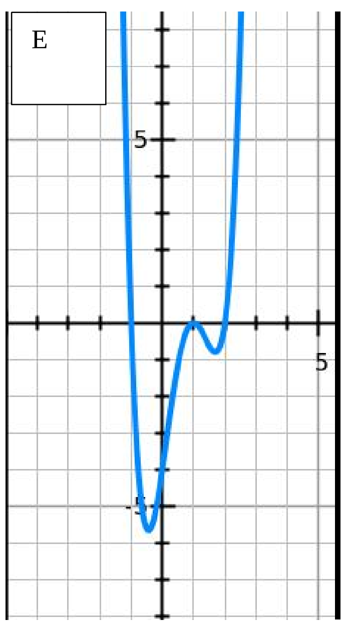
$y = 2(x-2)(x+1)^2(x-1)^2$ Answer: F

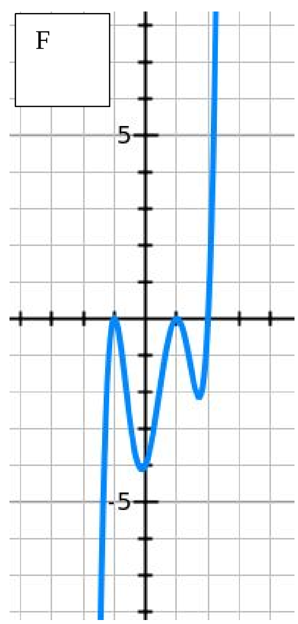








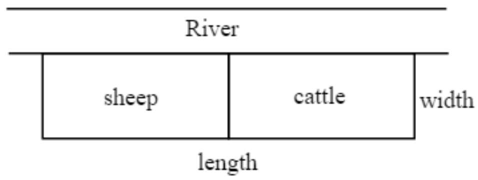




Part III. There are 6 problems in this section. <u>Choose any 4</u>.
Indicate in the boxes the problems you want graded. [10 points each]

❒ Grade

21. A rancher needs to enclose a rectangular corral. The corral will be divided into 2 sections as shown below. Assume the river forms one side of the corral and 60 ft. of fencing is available.



a) Write a function $A(x)$ for the total area of the corral.

Length: $60 - 3x$

$A(x) = x(60 - 3x)$

b) What dimensions yield maximum area?

$A(x) = -3x^2 + 60x$

$x = \frac{-b}{2a} = \frac{-60}{2(-3)} = 10$

$L = 60 - 3(10) = 30$

Dimensions are: 10ft by 30ft

❒ Grade

22. The sales S, of a product have declined in recent years. Assuming sales are decreasing according to the exponential decay model, $S(t) = 75e^{-0.02t}$ million. Determine the time it would take for the sales to reach 50 million. Leave your answer in exact form since no calculators are allowed.

Set up: $50 = 75e^{-.02t}$

Solve for t:

$\frac{50}{75} = e^{-.02t}$ (Divide by 75)

$\frac{2}{3} = e^{-.02t}$ (Reduce Fraction)

$\ln\left(\frac{2}{3}\right) = \ln(e^{-.02t})$ (Take ln of both sides)

$\ln\left(\frac{2}{3}\right) = -.02t$ (Simplify log on right)

$$t = \frac{\ln\left(\frac{2}{3}\right)}{-.02}$$

23. Given the polynomial $f(x) = x^3 + x^2 - 2$, find all zeros (real and complex).

❐ Grade

Find 1 rational zero:

$f(1) = 1 + 1 - 2 = 0 \rightarrow \ 1 \ is\ a\ zero$

$$\begin{array}{r|rrrr} 1 & 1 & 1 & 0 & -2 \\ & & 1 & 2 & 2 \\ \hline & 1 & 2 & 2 & 0 \end{array}$$

$x^2 + 2x + 2 = 0$

doesn't factor

$$x = \frac{-2 \pm \sqrt{4 - 4(2)}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$$

$\boxed{\text{3 zeros are: } 1,\ 1+i,\ 1-i}$

24. Solve for x. Be sure to check your solutions!

❐ Grade

$3 + \sqrt{3x + 1} = x$

$$\sqrt{3x + 1} = x - 3$$

$$3x + 1 = (x - 3)^2 = x^2 - 6x + 9$$

$$0 = x^2 - 9x + 8$$

$$0 = (x - 8)(x - 1)$$

$$x = 1, 8$$

Check both:

Right: $x = 1$

Left: $3 + \sqrt{3(1) + 1} = 3 + \sqrt{4} = 5$

$1 \neq 5$ so $x = 1$ is not really a solution

Right: $x = 8$

Left: $3 + \sqrt{3(8) + 1} = 3 + \sqrt{25} = 8$

Solution: $\underline{x = 8}$

25. Let $f(x) = -x^2 + x$. Set up and simplify the difference quotient: $\frac{f(x+h)-f(x)}{h}$

❐
Grade

$$-(x+h)^2 + (x+h) - (-x^2+x)$$
$$= -x^2 - 2xh - h^2 + x + h + x^2 - x$$
$$= -2xh - h^2 + h \ (top)$$
$$= \frac{-2xh - h^2 + h}{h} = -2x - h + 1$$

26. The endpoints of the diameter of a circle are $(3, 0)$ and $(1, -4)$. Find the

a) Center of the circle

b) Radius of the circle

❐
Grade

Midpoint Formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\left(\frac{3+1}{2}, \frac{0-4}{2}\right) = (2, -2)$

Distance Formula:

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(3-1)^2 + (0+4)^2}$$
$$= \sqrt{20} = 2\sqrt{5}$$
$$r = \frac{d}{2} = \sqrt{5}$$