Part I. Answer all problems in this section. Little partial credit will be given. [6 points each]

1. Write the following inequalities in interval notation:
   a. \( x \geq 0 \) \[ (0, \infty) \]
   b. \(-2 < x \leq 5\) \[ (-2, 5] \]
   c. \( x \neq 10 \) \[ (-\infty, 10) \cup (10, \infty) \]

2. Let \( f(x) = 3x - 2 \) and \( g(x) = x^2 + 2 \). Find and simplify
   a. \((f - g)(1) = f(1) - g(1) = [3(1) - 2] - [(1)^2 + 2] = 3 - 2 - 1 - 2 = -2\)
   b. \((f \circ g)(x) = f(g(x)) = f(x^2 + 2) = 3(x^2 + 2) - 2 = 3x^2 + 6 - 2 = 3x^2 + 4\)

3. Simplify the following to \(a + bi\) form.
   a. \((1 + 2i)(3 - i) = 3 + 6i - i - 2i^2 = 3 + 5i + 2 = 5 + 5i\)
   b. \((2i)^3 = 8i^3 = -8i\)

4. Evaluate:
   a. \( \ln e^{2x} = 2x - 1 \)
   b. \( \log 1 = 0 \)
   c. \( \log_3 9 = 2 \)
5. Given the piecewise function:

\[ f(x) = \begin{cases} 
3x - 1, & \text{for } x > 2 \\
-1, & \text{for } x \leq 2 
\end{cases} \]

Fill in the table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>( 3(3) - 1 = 8 )</td>
</tr>
</tbody>
</table>

6. Write an equation for a function that has a graph with the given characteristics:
   a. The shape of \( y = x^2 \) but upside-down and shifted right 7 units.

   \[ y = -(x - 7)^2 \]

   b. The shape of \( y = \sqrt{x} \) but reflected across the y-axis.

   \[ y = \sqrt{-x} \]

7. Solve for \( x \):

   \[ |3x + 4| - 2 = 10 \]

   \[ |3x + 4| = 12 \]

   \[ 3x + 4 = 12 \quad \text{and} \quad 3x + 4 = -12 \]

   \[ x = \frac{8}{3} \quad \text{and} \quad x = -\frac{16}{3} \]

8. Solve for \( x \):

   \[ x^2 + 25 = 0 \]

   \[ x = \pm 5i \]
9. Thara’s T-shirt store sold 36 shirts one day. All blue T-shirts cost $12 each and all green shirts cost $18 each. Total receipts for the day were $522. How many of each color of shirt were sold?

Set up a system – but you do not need to solve!

\[ b + g = 36 \]
\[ 12b + 18g = 522 \]

10. Find a 3rd degree polynomial (in polynomial form) with zeros: 2 and \(-5i\).

Write the polynomial on the line below.

\(-5i\) is a zero implies that \(5i\) is also a zero

\[ f(x) = (x - 2)(x + 5i)(x - 5i) \]
\[ f(x) = (x - 2)(x^2 + 25) \]
\[ f(x) = x^3 - 2x + 25x - 50 \]
Part II. Answer all problems in this section. [10 points each]
Some partial credit will be given based on work shown.

11. State the slope of the following lines:
   a. \( x + 2y = 9 \)
      
      \[ 2y = -x + 9 \]
      \[ y = \frac{-1}{2}x + \frac{9}{2} \]
      
      \( m = \frac{-1}{2} \)

   b. \( 2x = -5 \)
      
      \( y = -\frac{4}{5} \)

12. Graph the following lines:
   a. \( y = 4 \)
      
      \( y-intercept: (0, 4) \)

   b. \( y = -2x - 1 \)
      
      \( y-intercept: (0, -1) \)

13. Given \( f(x) = x^2 - 2x - 3 \), find the following and sketch the graph. Be sure to label all intercepts and the vertex.

   Vertex: \((\_\_, \_\_\_\_, -4)\)
   
   \[ \frac{-(-2)}{2(1)} = 1 \]
   
   \[ f(1) = 1 - 2 - 3 = -4 \]

   Zeros:
   
   \[ x^2 - 2x - 3 = 0 \]
   
   \[ (x - 3)(x + 1) = 0 \]

   \( y-intercept: (0, -3) \)
14. Given a function \( f(x) \), find the inverse function \( f^{-1}(x) \). Show all work algebraically and circle your answers.

a. \( f(x) = \sqrt[3]{2x + 3} \)

\[
x = \sqrt[3]{2y + 3} \\
x^3 = 2y + 3 \\
\frac{x^3 - 3}{2} = y \\
f^{-1}(x) = \frac{x^3 - 3}{2}
\]

b. \( f(x) = 5x^3 - 8 \)

\[
x = 5y^3 - 8 \\
x + \frac{8}{5} = y^3 \\
\frac{3}{5} \sqrt{x + \frac{8}{5}} = y \\
f^{-1}(x) = \frac{3}{5} \sqrt{x + \frac{8}{5}}
\]

15. Give the following intervals, in interval notation, based on the function \( f(x) \) shown in the graph.

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Domain of ( f ): ((-\infty, \infty))</td>
<td>b. Range of ( f ): ([-1, \infty))</td>
</tr>
<tr>
<td>c. ( f ) is increasing: ((2, \infty))</td>
<td>d. ( f ) is constant: ((-2, 2))</td>
</tr>
<tr>
<td>e. ( f(x) \leq 0 ) ([-3, 3])</td>
<td></td>
</tr>
</tbody>
</table>

16. Solve for \( x \).

a. \( 8^{x-1} = \frac{1}{2} \)

\[
(2^3)^{x-1} = 2^{-1} \\
3(x - 1) = -1 \\
3x - 3 = -1 \\
x = \frac{2}{3}
\]

b. \( 2e^{x+5} + 3 = 9 \)

\[
e^{x+5} = \frac{9 - 3}{2} = 3 \\
\ln e^{x+5} = \ln 3 \\
x + 5 = \ln 3 \\
x = \ln 3 - 5
\]
17. Find the following and sketch the graph of \( f(x) = \frac{-x}{x-4} \). Be sure to label all asymptotes and intercepts and use additional points as needed.

Zeros: 
\[-x = 0 \quad \Rightarrow \quad x = 0\]

y-int: 
\[f(0) = \frac{0}{-4} = 0\]

HA: 
\[-\frac{x}{x} = -1\]

VA: 
\[x = 4\]

y = -1

18. Solve the following inequalities. State your answers in interval notation.

a. \[x^2 + 3x - 28 < 0\]
   \[(x + 7)(x - 4) = 0\]
   \[x = -7, 4\]

b. \[-3 \leq \frac{x+5}{2} < 14\]
   \[-6 \leq x + 5 < 28\]
   \[-11 \leq x < 23\]
   \([-11, 23]\)

19. Solve the following system. Write your answer as an ordered pair.
\[
\begin{align*}
5x + y &= -20 \\
7x - 3y &= -50
\end{align*}
\]

Multiply eq1 by 3:
\[
eq 15x + 3y = -60
\]

Subtract eq2:
\[
15x + 3y = -60 \quad \text{and} \quad 7x - 3y = -50
\]

\[22x = -110\]

\[x = -\frac{110}{22} = -5\]

\[5(-5) + y = -20 \Rightarrow y = 5\]

Solution: \((-5, 5)\)
20. Match the equation with the graph below. Put letter choice in answer blank.

\[ y = 2(x - 2)(x + 1)(x - 1) \]  \hspace{1cm} \text{Answer: A}

\[ y = -2(x - 2)^2(x + 1)(x - 1) \]  \hspace{1cm} \text{Answer: C}

\[ y = 2(x - 2)(x + 1)^2(x - 1)^2 \]  \hspace{1cm} \text{Answer: F}
21. A rancher needs to enclose a rectangular corral. The corral will be divided into 2 sections as shown below. Assume the river forms one side of the corral and 60 ft. of fencing is available.

<table>
<thead>
<tr>
<th>River</th>
<th>sheep</th>
<th>cattle</th>
<th>width</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Write a function $A(x)$ for the total area of the corral.

Length: $60 - 3x$

$A(x) = x(60 - 3x)$

b) What dimensions yield maximum area?

$A(x) = -3x^2 + 60x$

$x = \frac{-b}{2a} = \frac{-60}{2(-3)} = 10$

$L = 60 - 3(10) = 30$

Dimensions are: 10ft by 30ft

22. The sales $S$, of a product have declined in recent years. Assuming sales are decreasing according to the exponential decay model, $S(t) = 75e^{-0.02t}$ million. Determine the time it would take for the sales to reach 50 million. Leave your answer in exact form since no calculators are allowed.

Set up: $50 = 75e^{-0.02t}$

Solve for $t$:

$\frac{50}{75} = e^{-0.02t}$ (Divide by 75)

$\frac{2}{3} = e^{-0.02t}$ (Reduce Fraction)

$\ln\left(\frac{2}{3}\right) = \ln(e^{-0.02t})$ (Take ln of both sides)

$\ln\left(\frac{2}{3}\right) = -0.02t$ (Simplify log on right)

$t = \frac{\ln\left(\frac{2}{3}\right)}{-0.02}$
23. Given the polynomial \( f(x) = x^3 + x^2 - 2 \), find all zeros (real and complex).

Find 1 rational zero:

\[
f(1) = 1 + 1 - 2 = 0 \rightarrow 1 \text{ is a zero}
\]

\[
\begin{array}{c|ccc}
 & 1 & 0 & -2 \\
\hline
1 & 1 & 0 & 2 \\
 & 1 & 2 & 0
\end{array}
\]

\( x^2 + 2x + 2 = 0 \) doesn't factor

\[
x = \frac{-2 \pm \sqrt{4 - 4(2)}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i
\]

24. Solve for \( x \). Be sure to check your solutions!

\[
3 + \sqrt{3x + 1} = x
\]

\[
\sqrt{3x + 1} = x - 3
\]

\[
3x + 1 = (x - 3)^2 = x^2 - 6x + 9
\]

\[
0 = x^2 - 9x + 8
\]

\[
0 = (x - 8)(x - 1)
\]

\[
x = 1, 8
\]

Check both:

Right: \( x = 1 \) \hspace{1cm} \text{Right: } x = 8

Left: \( 3 + \sqrt{3(1) + 1} = 3 + \sqrt{4} = 5 \) \hspace{1cm} \text{Left: } 3 + \sqrt{3(8) + 1} = 3 + \sqrt{25} = 8

1 ≠ 5 \hspace{0.5cm} \text{so } x = 1 \text{ is not really a solution}

Solution: \( x = 8 \)
25. Let \( f(x) = -x^2 + x \). Set up and simplify the difference quotient:

\[
\frac{f(x+h) - f(x)}{h}
\]

\[
-(x + h)^2 + (x + h) - (-x^2 + x) = -x^2 - 2xh - h^2 + x + h + x^2 - x = -2xh - h^2 + h \text{ (top)}
\]

\[
\frac{-2xh - h^2 + h}{h} = -2x - h + 1
\]

26. The endpoints of the diameter of a circle are \((3, 0)\) and \((1, -4)\). Find the

a) Center of the circle

b) Radius of the circle

Midpoint Formula: \(\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\)

Distance Formula:

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
= \sqrt{(3 - 1)^2 + (0 + 4)^2} = \sqrt{20} = 2\sqrt{5}
\]

\[
r = \frac{d}{2} = \sqrt{5}
\]