Part I. Problems in this section are mostly short answer and multiple choice. Little partial credit will be given. 4 points each.

1. Factor completely.
   a) \(2x^3 - 8x^2 + 6x\)
   \[2x(x^2 - 4x + 3)\]
   \[2x(x - 3)(x - 1)\]
   b) \(x^3 + 5x^2 - 4x - 20\)
   \[x^2(x + 5) - 4(x + 5)\]
   \[(x^2 - 4)(x + 5)\]
   \[(x - 2)(x + 5)(x + 5)\]

2. Find the domain of the function \(g(x) = \frac{1}{\sqrt{x - 3}}\).
   a) \((-\infty, \infty)\)
   b) \((-\infty, 3)\)
   c) \((-\infty, 3]\)
   d) \([3, \infty)\)
   e) \([3, \infty)\)

3. Use properties of logs to express as a single log.
   \(2\log_a x - 3\log_a(x + 5)\)
   \(\log_a x^2 - 3\log_a(x + 5)\)
   \(\log_a \frac{x^2}{(x + 5)^3}\)

4. Complete the following table:

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (x \leq 4)</td>
<td>((-\infty, 4])</td>
</tr>
<tr>
<td>b. (2 &lt; x &lt; 5)</td>
<td>((2, 5])</td>
</tr>
<tr>
<td>c. (</td>
<td>x</td>
</tr>
<tr>
<td>d. (</td>
<td>x</td>
</tr>
</tbody>
</table>
5. Let \( f(x) = 4x - 3 \) and \( g(x) = x^2 + 1 \).

   a) Find and simplify \((f - g)(-2)\).

   \[
   f(-2) = 4(-2) - 3 = -11
   \]

   \[
   g(-2) = (-2)^2 + 1 = 5
   \]

   \[
   (f - g)(-2) = -11 - 5 = -16
   \]

   b) Find and simplify \((f \circ g)(x)\).

   \[
   f(x^2 + 1) = 4(x^2 + 1) - 3 = 4x^2 + 4 - 3 = 4x^2 + 1
   \]

6. Solve for \( x \):

   \[
   \frac{2x + 1}{x - 4} = \frac{4x + 1}{x - 2}
   \]

   Cross-multiplication:

   \[
   (2x + 1)(x - 2) = (4x + 1)(x - 4)
   \]

   \[
   2x^2 - 4x + x - 2 = 4x^2 + 1x - 16x - 4
   \]

   \[
   2x^2 - 3x - 2 = 4x^2 - 15x - 4
   \]

   \[
   -2x^2 + 12x + 2 = 0
   \]

   

7. Find the equation of each line.

   \[
   \ell_1 : y = \frac{4}{3}x
   \]

   \[
   \ell_2 : y = 4
   \]

\[\ell_1: \quad \ell_2: \]
8. Given the graph of \( f(x) = \frac{x+1}{x-2} \), state all \( x \) such that
   
   a) \( f(x) \) is decreasing
      \((-\infty, 2) \cup (2, \infty)\)
   
   b) \( f(x) \geq 0 \)
      \((-\infty, -1] \cup [2, \infty)\)

9. Solve for \( x \):
   \[
   \frac{7}{x+4} - \frac{3}{x} = \frac{5}{x^2+4x}
   \]
   
10. Solve:
   \[
   |2x-3|-4 = 1
   \]

11. Graph each function. Label all intercepts and asymptotes.
   
   \( f(x) = e^x - 2 \)
   
   \( g(x) = \log_2(-x) \)

   intercept: \((0, -1)\)
   equation of asymptote: \(y = -2\)

   intercept: \((-1, 0)\)
   equation of asymptote: \(x = 0\)
12. State the vertex of each.

a) \( y = x^2 \)  \[ v = (0, 0) \]

b) \( y = (x - 2)^2 \)  \[ v = (2, 0) \]

c) \( y = (x - 2)^2 + 3 \)  \[ v = (2, 3) \]

d) \( y = -(x - 2)^2 + 3 \)  \[ v = (2, 3) \]

Part II. There are 10 problems in this section. Partial credit will be awarded. Show all work. 12 pts. each.

13. Solve: \( x^2 - 2x + 4 = 0 \) (Include real and complex solutions.)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2} = \frac{2 \pm \sqrt{-12}}{2} = 1 \pm \sqrt{3}i
\]

Solution(s): \( x = 1 \pm \sqrt{3}i \).

14. Given the function \( f(x) = -x^2 + 5x + 14 \)

a) State the zeros of the function.

\( (7, 0), (2, 0) \)

b) State the concavity of \( f \)

d) Maximum/minimum value = \( \frac{81}{4} \).
15. Solve for $x$.
   a) $9^{3x-1} = \frac{1}{27}$
   
   
   b) $e^{3x+1} = 5$
   
   
16. Given the function $f(x) = -(x + 3)^2(2x - 5)(x - 1)$

   a) find $y$-intercept.
   
   b) find zeros and state their multiplicities.
   
   c) Is $f(x)$ tangent to the $x$-axis?
   If so, where? $x = 3$

   d) Sketch graph. Label all intercepts.

17. Find a formula for the inverse given $f(x) = \frac{2x^3 - 1}{5}$.

   
   $f^{-1}(x) = \sqrt[3]{\frac{x + 1}{2}}$
18. Given the polynomial \( g(x) = x^3 + 4x^2 - 9 \)
   
   a) State all possible rational zeros.
   \[ \pm \frac{1}{3} \]

   b) Find all zeros (real or complex.)
   \[ x = -3, -1 \pm \sqrt{13} \]
   \[ x = -3, -1 \pm \sqrt{13} \]

19. Find all asymptotes, x-intercepts, and y-intercepts for the graph \( f(x) = \frac{2x-1}{x+3} \).
   
   a) The equation of the vertical asymptote(s) is/are \( x = -3 \).
   
   b) The equation of the horizontal asymptote(s) is/are \( y = 2 \).
   
   c) The x-intercept is at the point \( (k, 0) \).
   
   d) The y-intercept is at the point \( (0, -\frac{1}{3}) \).
   
   e) Sketch the graph of \( f(x) \). Label all intercepts and asymptotes.
20. Find a 3rd degree polynomial with real coefficients whose zeros are -2 and 2i.

\((x+2)(x^2+4)\)

\((x+2)(x-2i)(x+2i)\)

\(x^3 + 3x^2 + 4x + 8\)

21. Solve \(3 + \sqrt{3x+1} = x\). Check all solutions.

\(3x + 1 = (x - 3)^2\)

\(3x + 1 = x^2 - 6x + 9\)

\(3x + 1 = x^2 - 6x + 9\)

\(0 = x^2 - 3x + 8\)

\(0 = (x - 4)(x - 2)\)

\(x = 4\) \(\checkmark\)

\(x = 2\) \(\text{No}\)

22. Solve for \(x\): \(\log_2 x + \log_2 (x + 2) = 3\)

\(\log_2 x (x + 2) = 3\)

\(6 = x^2 + 2x\)

\(0 = x^2 + 2x - 6\)

\(0 = (x + 4)(x - 2)\)

\(x = -4\) \(\text{No}\)

\(x = 2\) \(\checkmark\)

\(x = 2\) \(\checkmark\)
Part III. There are 6 problems in this section. Choose any 4. Indicate in the boxes the problems you want graded. 8 points each.

☐ Grade 23. State the equation of a circle passing through (1, 4) with the center (-2, 3).

\[(x + 2)^2 + (y - 3)^2 = r^2\]

\[(x + 2)^2 + (y - 3)^2 = 10\]

☐ Grade 24. Strontium 90 is radioactive material that decays according to the equation \[A = A_0e^{-0.0244t}\]

where \(A_0\) is the initial amount present and \(A\) is the amount present at time \(t\) (in years). If the initial amount is 50 grams what is the half-life of Strontium 90? i.e. how long will it take for the 50 grams to be reduced by half? Leave your answer in exact form (in terms of logarithms).

\[t = \frac{\ln 2}{-0.0244}\]
25. A stone is thrown directly upward. The height of the stone \( t \) seconds after it has been thrown is given by \( s(t) = -16t^2 + 64t \) ft. Find the time it takes for the stone to reach a height of 28 ft.

\[
28 = -16t^2 + 64t
\]

\[
0 = -16t^2 + 64t - 28
\]

\[
= 4t^2 - 16t + 7
\]

\[
= (2t-1)^2 + 7
\]

\[
t = \frac{1}{2}
\]

\[
t < \frac{3}{2}
\]

26. Solve \( \frac{2x-1}{x+4} < 0 \). Express in interval form.

Grade

\[x = \frac{1}{2}\]

\[x = -4\]

\[f(-3) = \frac{-1}{-1} = 1\]

\[f(0) = \frac{-1}{4} < 0\]

\[f(1) = \frac{1}{1} < 0\]
27. Graph the following: \( f(x) = \begin{cases} 2x - 3, & x < 0 \\ x, & x \geq 0 \end{cases} \)

28. A rectangular picture frame measures 9 in. by 12 in. The thickness of the frame is the same on all 4 sides. Call this thickness \( x \) inches.

a) Find a function for the area of the picture in terms of \( x \).

\[ A(x) = (12 - 2x)(9 - 2x) \]

b) Suppose the area of the picture measures 54 sq. in. Find \( x \).

\[ 54 = (12 - 2x)(9 - 2x) \]

\[ 54 = 2(6 - x)(9 - 2x) \]

\[ 27 = 54 - 21x + 4x^2 \]

\[ 0 = 2x^2 - 71x + 27 \]

\[ 0 = (2x - 3)(x - 9) \]

\[ x = \frac{3}{2}, 9 \]

BE SURE YOU HAVE MARKED THE 4 PROBLEMS TO BE GRADED.