1. (12 pts) Evaluate each of the following. Show all work. Simplify your answers to an integer.

(a) $6!$

(b) $C(8, 4)$

(c) $P(12, 3)$

2. (15 pts) 320 people are surveyed about vitamins: Vitamin $B$ (energy), Vitamin $C$ (protection), and Vitamin $E$ (stamina). It is found that 52 people took all 3 Vitamins; 150 took Vitamin $B$, 200 took Vitamin $C$, 165 took Vitamin $E$, 57 took Vitamins $B$ and $C$, 125 took Vitamins $B$ and $E$, and 82 took vitamins $C$ and $E$.

(a) Represent the data on the Venn diagram:

(b) How many people took none of the 3 Vitamins?

(c) How many people took Vitamins $B$ and $E$, but not Vitamin $C$?
3. (10 pts) Companies whose stocks are listed on the New York Stock Exchange have their company name represented by 2 or 3 letters. What is the maximum number of companies that can be listed on the New York Stock Exchange if:

(a) Repetition of letters is allowed.

(b) Repetition of letters is not allowed.

4. (12 pts) The following data show the number of voters in a sample of 500, categorized by their religion and their voter preference.

<table>
<thead>
<tr>
<th></th>
<th>Democrat, D</th>
<th>Republican, R</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catholic, C</td>
<td>60</td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>Jewish, J</td>
<td>20</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>Muslim, M</td>
<td>45</td>
<td>35</td>
<td>80</td>
</tr>
<tr>
<td>Protestant, P</td>
<td>170</td>
<td>100</td>
<td>270</td>
</tr>
<tr>
<td>Totals</td>
<td>295</td>
<td>205</td>
<td>500</td>
</tr>
</tbody>
</table>

(a) Find the probability a person is a Democrat.

(b) Find the probability a person is a Catholic.

(c) Find the probability a person is a Catholic, given that they are a Democrat.

(d) Are the events M and R independent?
5. (15 pts) John and Sue left their bags on the floor. John’s bag contains 6 yellow and 4 green tennis balls. Sue’s bag contains 3 yellow and 8 green tennis balls. A thief opens a bag at random and steals 1 tennis ball.

(a) Draw a tree diagram representing this data, listing all outcomes and their probabilities.

(b) What is the probability that the stolen ball is yellow?

(c) If the stolen ball is green, what is the probability it was taken from Sue’s bag?

6. (9 pts)
(a) How many ways are there to arrange 7 different books on a shelf?

(b) How many 7-letter words (real or imaginary) can be made from the word

S U C C E S S?
For the problems on this page (problems 7 and 8) you may leave your answers expressed in terms of products, powers, permutations and/or combinations. You need not work out the exact answer.

7. (15 pts) A salesperson contacts prospective customers by telephone, and estimates that 25% of all telephone calls results in a sale. The salesperson makes 8 telephone calls. Find the probability that

(a) All 8 telephone calls result in a sale.

(b) None of the 8 telephone calls result in a sale.

(c) At least 2 of the 8 telephone calls result in a sale.

8. (12 pts) A “hand” of 5 cards is dealt from a regular deck of 52 cards. Find the probability that

(a) The hand contains only red cards.

(b) The hand contains the Jack of Diamonds.

(c) The hand contains exactly 2 Jacks.
9. (14 pts) 
(a) Find the equation of the line passing through \((4, -3)\) and \((2, 1)\). Write the answer in slope-intercept form, \(y = mx + b\).

(b) Compute the \(x-\) and \(y-\)intercepts of this line.

(c) Determine whether the two lines \(x + 4y = 3\) and \(2x + 8y = 6\) are intersecting, parallel, or coincident.

10. (10 pts) Given the two matrices \(A = \begin{bmatrix} 1 & 1 \\ 3 & 7 \end{bmatrix}\) and \(B = \begin{bmatrix} 2 & 1 & \frac{3}{2} \\ 3 & -1 & 4 \end{bmatrix}\).

Find

(a) the inverse matrix \(A^{-1}\)

(b) \(AB\)
11. (14 pts) Solve the system of linear equations by finding the reduced row-echelon form of the augmented matrix. Label all row operations. Clearly state your final answer.

\[
\begin{align*}
x + z &= 1 \\
y + 2z &= 7 \\
2x + y + 3z &= 7
\end{align*}
\]
12. (12 pts) The following augmented matrices represent systems of linear equations in variables $x$, $y$ and $z$. In each case either state the general solution or that no solution exists.

(a) \[
\begin{bmatrix}
1 & 0 & 0 & 2 \\
0 & 1 & 0 & 4 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
1 & 0 & 2 & 1 \\
0 & 1 & 3 & 3 \\
0 & 0 & 0 & -5
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
1 & 2 & 0 & 2 \\
0 & 0 & 1 & 8 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

13. (12 pts) Formulate as an LP model. DO NOT ATTEMPT TO SOLVE IT.

A company makes rackets for tennis and squash. Each tennis racket requires 3 units of aluminum and 1 unit of cord. Each squash racket requires 1.6 units of aluminum and 0.8 units of cord. The company has 2000 units of aluminum and 700 units of cord available. The company is not able to manufacture more than 500 rackets in total, and must manufacture more tennis rackets than squash rackets. The profit on each tennis racket is $6, and on each squash racket is $8. How many rackets of each type should the company make in order to maximize profit?
14. (12 pts) Consider the following linear programming problem

Maximize \( P = 4x + y \)

subject to the constraints

\[
\begin{align*}
3x - y & \geq 6 \\
x + y & \leq 6 \\
x & \geq 0, \ y & \geq 0
\end{align*}
\]

(a) Sketch the feasible region labeling all corner points.

(b) Solve the problem \textit{geometrically} and report the complete solution.
15. (12 pts) For each of the following tableaux, perform one of the following steps:
1) If a pivot is required, circle the pivot element and write ‘pivot needed’
   but DO NOT PIVOT.
2) If there is no optimal solution, say so and say why.
3) If the problem is finished, report the complete solution.

(a)

<table>
<thead>
<tr>
<th>BV</th>
<th>P</th>
<th>x₁</th>
<th>x₂</th>
<th>s₁</th>
<th>s₂</th>
<th>RHS</th>
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<tr>
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(b)

<table>
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<th>P</th>
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<th>x₂</th>
<th>s₁</th>
<th>s₂</th>
<th>RHS</th>
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<td>0</td>
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<tr>
<td>s₂</td>
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(c)

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<th>x₂</th>
<th>x₃</th>
<th>s₁</th>
<th>s₂</th>
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<tbody>
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<tr>
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<td>-3</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>17</td>
</tr>
</tbody>
</table>
16. (14 pts) Use the simplex method to solve the following LP model:

Maximize \( P = 3x_1 + x_2 \)

subject to the constraints

\[
\begin{align*}
  x_1 + 3x_2 & \leq 5 \\
  x_1 - x_2 & \leq 1 \\
  x_1 & \geq 0, \; x_2 & \geq 0
\end{align*}
\]