1. Use the graph of \( y = f(x) \) below to estimate each limit if it exists. If the limit does not exist, write “DNE”.

(a) \( \lim_{x \to 1^-} f(x) = \) ______

(b) \( \lim_{x \to 1} f(x) = \) ______

(c) \( \lim_{x \to -1} f(x) = \) ______

(d) \( \lim_{x \to -3^+} f(x) = \) ______

2. Find each limit, if it exits. If the limit does not exist, write “DNE”.

(a) \( \lim_{x \to \infty} \frac{x^3 + x}{6 + 5x^2 - x^3} \)

(b) \( \lim_{x \to -2} \frac{x^2 - 2x - 8}{x^2 - 4} \)
3. Find the derivative of each function. Do not simplify.

(a) \( f(x) = \frac{x^5}{10} - 6\sqrt{x} + 11 \)

(b) \( f(x) = \frac{x^2 - 2x}{3x + 5} \)

(c) \( f(x) = (7 - x)\ln(x^2 + 9) \)

4. Find an equation of the tangent line to the curve \( y = \sqrt{3 + 2x - x^2} \) at the point \((0, \sqrt{3})\).
5. Match each graph below to one of the descriptions.

(a) _______ \( f'(x) < 0 \) and \( f''(x) > 0 \) on \((0, \infty)\)

(b) _______ \( f'(x) > 0 \) and \( f''(x) > 0 \) on \((0, \infty)\)

(c) _______ \( f'(x) < 0 \) and \( f''(x) < 0 \) on \((0, \infty)\)

(d) _______ \( f'(x) > 0 \) and \( f''(x) < 0 \) on \((0, \infty)\)

6. Cycles Inc. determines that in order to sell \( x \) bicycles, the price per bicycle must be

\[ p(x) = 300 - 0.1x. \]

It also determines that the total cost of producing \( x \) bicycles is given by

\[ C(x) = 180 + 0.2x^2. \]

How many bicycles should this company produce and sell in order to maximize profit? (Recall that the total revenue is \( R(x) = xp(x) \).)
7. Given that \( f(x) = \frac{12x}{x^2 + 3} \), \( f'(x) = -\frac{12(x^2 - 3)}{(x^2 + 3)^2} \) and \( f''(x) = \frac{24x(x^2 - 9)}{(x^2 + 3)^3} \), answer the questions below.

(a) Find the asymptote(s).

(b) Find the interval(s) over which \( f(x) \) is increasing/decreasing.

(c) Find all points where relative maxima and minima occur.

(d) Find the interval(s) over which \( f(x) \) is concave up/concave down.

(e) Find all points of inflection.

(f) Use parts (a) through (e) to sketch \( y = f(x) \). Mark each asymptote using a dashed line.
8. Compute each integral.

(a) \( \int \left( 6x^3 - \frac{3}{x^2} + 10 \right) dx \)

(b) \( \int_{0}^{\sqrt{3}} x\sqrt{x^2 + 1} \, dx \)

(c) \( \int_{1}^{e} \frac{(\ln x)^2}{x} \, dx \)

(d) \( \int xe^{-x} \, dx \) (Use integration by parts.)
9. Hugo Ltd. estimates that its sales $S(t)$, in dollars, of apparel will grow continuously at a rate

$$S'(t) = 50e^{2t},$$

where $t$ is in days. Find the accumulated sales from the 2nd day through the 6th day, i.e. from $t = 1$ to $t = 6$.

10. Find the area of the region enclosed by the graphs of $y = x^2 - 2$ and $x + y = 0$. 

![Graph of the region enclosed by the curves $y = x^2 - 2$ and $x + y = 0$.]
11. Given \( f(x, y) = x^3 e^{-y^2} + 3x^2 y - 9y^5 \), find:

(a) \( f_y \)

(b) \( f_{yy} \)

12. A one-product company finds that its profit \( P \), in millions of dollars, is given by

\[
P(x, y) = 210 - x^2 y + 2xy + 35y - 2y^2,
\]

where \( x \) is the amount spent on advertising, in millions of dollars, and where \( y \) is the price per item of the product, in dollars. The critical points of the function \( P \) are \((1,9)\) and \((7,0)\). Classify each critical point as a relative maximum, a relative minimum, or a saddle point.
13. Use the method of Lagrange multipliers to find the maximum value of the function

\[ f(x, y) = 3xy, \]

where \( x > 0 \) and \( y > 0 \), subject to the constraint

\[ x^2 + 3y = 3. \]