1. (15 points) Find the following limits. If a limit does not exist write ‘DNE’.

a) $\lim_{x\to 0} \frac{x^2 - x}{x}$

b) $\lim_{x\to 3} \frac{x^2 + 1}{x - 3}$

c) $\lim_{x\to \infty} \frac{\sqrt{x^2 + x - 4}}{x + 3}$

2. (24 points) Find the following derivatives. (Do not simplify.)

a) $f(x) = 3e^{x^2} + x^4 + (2x+1)^9$

b) $f(x) = x^3 e^{\sqrt{x}}$

c) $f(x) = \ln(x^3 + 8x + 1)$

d) $f(x) = \dfrac{(x^2 + 8x + 3)}{(2x+1)}$

3. (15 points) Given $f(x) = \dfrac{4x+8}{x-3}$. Find (if any)

a) The x-intercept(s) and the y-intercept(s).

b) The vertical asymptotes (VA) and the horizontal asymptotes (HA).

c) The derivative $f'(x)$.

d) Intervals where $f(x)$ is increasing and decreasing.

e) Put all this information on an accurate sketch of $f(x)$.

4. (12 points) Find the intervals where $f(x) = 2x^3 - 9x^2 + 27$ is concave up and concave down, and find any inflection points. (Do not sketch.)

5. (12 points) Find the absolute minimum and absolute maximum of $f(x) = xe^x$ on $[-2, 0]$. (Show all work.)

6. (12 points) Find the equation of the tangent line to the curve $y = \sqrt{3 + 2x - x^2}$ at the point $(0, \sqrt{3})$. Write your answer in slope-intercept $y = mx + b$ form.

7. (20 points) Evaluate the integrals

a) $\int e^x + \sqrt{x} - 8 + 3x^2 \; dx$

b) $\int x^5 \ln x \; dx$ (Use integration by parts.)

8. (24 points) Evaluate the definite integrals

a) $\int_0^2 x\sqrt{2x^2+1}\,dx$

b) $\int_1^e \frac{(\ln x)^2}{x}\,dx$

9. (10 points) The *total* profit (in dollars) from the sale of x watches is given by

$$P(x) = 12x - 0.03x^2 - 300.$$

a) Find the *marginal* profit function.

b) What is the number of watches that the company must produce and sell in order to maximize *total* profit.

10. (12 points) Sketch the area bounded between the two curves

$$f(x) = -x \quad \text{and} \quad g(x) = 2x - x^2$$

and then find this area.

11. (12 points) $S(x)$ is the price, in dollars per unit, that producers are willing to accept for x units of an item, and $D(x)$ is the price, in dollars per unit, that consumers are willing to pay for x units. Find the equilibrium quantity and equilibrium price.

$$S(x) = 60 - 2x^2 \qquad D(x) = x^2 + 9x + 30$$

12. (12 points) Given

$$f(x,y) = x^2y + 2x^2 + y^2 + 4.$$

The critical points of $f(x,y)$ are $(0,0),(2,-2)$ and $(-2,-2)$. identify them as relative minimum, relative maximum or saddle point.

13. (20 points) Use Lagrange multipliers to find the minimum value of the function

$$f(x, y) = 8xy + 4x^2 + 5y^2$$

subject to the constraint

$$2x + y = 40.$$