MATH 108 – REVIEW TOPIC 8

Linear Equations

Introduction. An equation is a statement of equality. It has two sides (separated by an equal sign) and at least one variable. Depending on the values assigned to each variable, the equation may or may not be true.

Illustration:

$2x + 5 = 9$ is true for $x = 2$ but false for all other values of x.

$x^2 - 3x - 4 = 0$ is true only when $x = 4$ or $x = -1$.

Check:
$$\left.\begin{array}{ll} \text{If } x = 4, & 4^2 - 3(4) - 4 = 0 \\ \text{If } x = -1, & (-1)^2 - 3(-1) - 4 = 0 \end{array}\right\} \text{ Both are true.}$$

Values that make an equation true are called solutions or roots.

I. SOLVING LINEAR EQUATIONS

In this section we will concentrate on solving linear (also called first degree) equations. Linear equations have at most one solution.

Exercise 1: Solve for x.

a) $8x - (3x - 2) = 10 - x$

b) $(x + 7)(x - 1) = (x + 1)^2$

c) $\dfrac{x+1}{3} - \dfrac{2x-5}{4} = 1$

When you have finished, click on and compare your steps with ours.

Answers

Here's a linear equation with an interesting twist.

Example: Solve $\dfrac{3x}{x-2} = 1 + \dfrac{6}{x-2}$.

Answer: Multiply each side by $(x - 2)$ to clear fractions.

$$\begin{aligned} 3x &= (x - 2) + 6 \\ x &= 2 \end{aligned}$$

Comment: The algebra is correct, but now you see why checking a solution is so important. Since division by 0 is undefined, $x = 2$ is not a solution. Therefore this equation has **"no solution"**.

II. SOLVING FOR AN INDICATED VARIABLE

Some linear equations, particularly formulas, contain several variables.

Illustration:

$$3x + 4y - 5 = 0$$

$$A = p + prt \qquad \text{(compound interest)}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{(electricity)}$$

The extra variables can be a distraction, but the steps used to solve any linear equation are very much the same.

Example: Solve $A = p + prt$ for r.

Answer:

$$A - p = prt$$

$$\frac{A-p}{pt} = r$$

Example: Solve $A = p + prt$ for p.

Answer: With two unlike terms containing p, only factoring allows you to get a single coefficient of p.

$$A = p(1 + rt)$$

$$\frac{A}{1+rt} = p$$

Exercise 2: Solve $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ for R_2. Answer

PRACTICE PROBLEMS for Topic 8

Solve for x and check.

8.1. $x(1+2x) = (2x-1)(x-2)$ Answer

8.2. $\dfrac{3}{2x-1} = \dfrac{5}{x+3}$ Answer

8.3. a) Solve $\dfrac{x+1}{x^2+2x} - \dfrac{x+4}{x^2+x} = \dfrac{-3}{x^2+3x+2}$ Answer

b) Simplify $\dfrac{x+1}{x^2+2x} - \dfrac{x+4}{x^2+x} + \dfrac{3}{x^2+3x+2}$ Answer

Solve for the indicated variable.

8.4. $x = \dfrac{y-1}{2y+3}$ for y Answer

8.5. $S = 2(lw + wh + lh)$ for l Answer

8.6. $A = \dfrac{1}{2}h(b_1 + b_2)$ for b_1 Answer

ANSWERS to PRACTICE PROBLEMS (Topic 8–Linear Equations)

8.1.

$$2x^2 + x = 2x^2 - 5x + 2$$
$$6x = 2$$
$$x = \frac{1}{3}$$

Check:

$$\frac{1}{3}\left[1 + 2\left(\frac{1}{3}\right)\right] = \left[2\left(\frac{1}{3}\right) - 1\right]\left[\frac{1}{3} - 2\right]$$
$$\frac{1}{3}\left(\frac{5}{3}\right) = \left(-\frac{1}{3}\right)\left(-\frac{5}{3}\right)$$
$$\frac{5}{9} = \frac{5}{9}; \quad x = \frac{1}{3}$$

is the solution.

Return to Problem

8.2. Using properties of proportions;

$$\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$$

Thus $\dfrac{3}{2x-1} = \dfrac{5}{x+3} \Rightarrow 3(x+3) = 5(2x-1)$

$$\Rightarrow x = 2$$

Check:

$$\frac{3}{2(2)-1} = \frac{5}{2+3}$$
$$1 = 1$$

Return to Problem

8.3. a)

$$\frac{x+1}{x(x+2)} - \frac{x+4}{x(x+1)} = \frac{-3}{(x+2)(x+1)} \qquad \text{LCD}=x(x+2)(x+1)$$

$$\begin{aligned}(x+1)(x+1)-(x+4)(x+2) &= -3x \qquad \text{Clearing fractions}\\ x^2+2x+1-(x^2+6x+8) &= -3x\\ -4x-7 &= -3x\\ x &= -7\end{aligned}$$

We'll leave the checking to you.

Return to Problem

8.3. b) This is not an equation. An expression cannot be solved and its denominators cannot be cleared (see Review Topic 5 on Rational Expressions).

$$\begin{aligned}&\frac{x+1}{x(x+2)} - \frac{x+4}{x(x+1)} + \frac{3}{(x+2)(x+1)}\\ &= \frac{(x+1)(x+1)}{x(x+2)(x+1)} - \frac{(x+4)(x+2)}{x(x+1)(x+2)} + \frac{3(x)}{(x+2)(x+1)x}\\ &= \frac{x^2+2x+1-(x^2+6x+8)+3x}{x(x+2)(x+1)}\\ &= \frac{-x-7}{x(x+2)(x+1)} \quad \text{or} \quad -\frac{x+7}{x(x+2)(x+1)}\end{aligned}$$

Return to Problem

8.4.

$$\begin{aligned}x(2y+3) &= y-1\\ 2xy-y &= -1-3x\\ y(2x-1) &= -(1+3x)\\ y &= -\frac{1+3x}{2x-1}\end{aligned}$$

There are other equivalent forms of this answer depending on signs. $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$.

Return to Problem

8.5.

$$\begin{aligned}S &= 2lh+2wh+2lh\\ S-2wh &= l(2w+2h)\\ \frac{S-2wh}{2(w+h)} &= l\end{aligned}$$

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8.6.
$$\begin{aligned} 2A &= hb_1 + hb_2 \\ 2A - hb_2 &= hb_1 \\ \frac{2A - hb_2}{h} &= b_1 \end{aligned}$$

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Question: What caused the need for factoring in 8.4 and 8.5, but not in 8.6? The unlike terms containing the variable (to be solved for) are the key.

Beginning of Topic 108 Skills Assessment

Solve for x.

a) $8x - (3x - 2) = 10 - x$

b) $(x + 7)(x - 1) = (x + 1)^2$

c) $\dfrac{x+1}{3} - \dfrac{2x-5}{4} = 1$

Answers:

a)
$$\begin{aligned} 8x - (3x - 2) &= 10 - x \\ 5x + 2 &= 10 - x \\ 6x &= 8 \\ x &= \frac{4}{3} \end{aligned}$$

Check :
$$\begin{aligned} 8\left(\frac{4}{3}\right) - \left[3\left(\frac{4}{3}\right) - 2\right] &= 10 - \frac{4}{3} \\ \frac{32}{3} - 2 &= 10 - \frac{4}{3} \\ \frac{26}{3} &= \frac{26}{3} \end{aligned}$$

Thus, $x = \dfrac{4}{3}$ is a solution or root.

b)
$$\begin{aligned} (x + 7)(x - 1) &= (x + 1)^2 \\ x^2 + 6x - 7 &= x^2 + 2x + 1 \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

Check :
$(9)(1) = 3^2$

Comment: This equation appears to be quadratic (x^2 terms are present). It becomes linear as you work through it.

c)
$$\begin{aligned} \frac{x+1}{3} - \frac{2x+5}{4} &= 1 \\ 12\left[\frac{x+1}{3} - \frac{2x+5}{4}\right] &= 12(1) \\ \cancel{12}\left(\frac{x+1}{\cancel{3}}\right) - \cancel{12}\left(\frac{2x+5}{\cancel{4}}\right) &= 12 \\ 4(x+1) - 3(2x+5) &= 12 \\ 4x + 4 - 6x - 15 &= 12 \\ -2x &= 23 \\ x &= -\frac{23}{2} \end{aligned}$$

Making use of the multiplicative property of equality (if $a = b$, $ac = bc$), multiply by 12 to clear fractions.

Return to Review Topic

Solve $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ for R_2.

Answer:

$$\left[\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}\right] RR_1R_2$$

$$\begin{aligned} R_1R_2 &= RR_2 + RR_1 && \text{Clearing fractions} \\ R_1R_2 - RR_2 &= RR_1 && \text{Collect terms containing } R_2 \\ R_2(R_1 - R) &= RR_1 \\ R_2 &= \frac{RR_1}{R_1 - R} \end{aligned}$$

Alternate Method:

$$\begin{aligned} \frac{1}{R} - \frac{1}{R_1} &= \frac{1}{R_2} \\ \frac{R_1 - R}{RR_1} &= \frac{1}{R_2} \\ \frac{RR_1}{R_1 - R} &= R_2 \quad \text{Equality of reciprocals:} \\ & \qquad\quad \text{if } a = b,\ \frac{1}{a} = \frac{1}{b} \end{aligned}$$

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