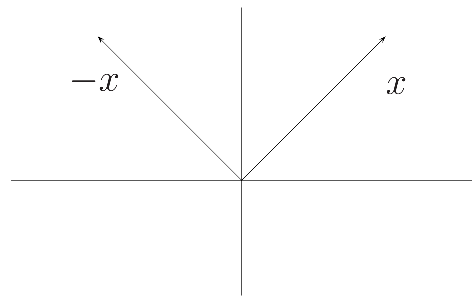
MATH 150 – TOPIC 2

PIECEWISE-DEFINED FUNCTIONS

I. Absolute Value Functions

Sometimes a function cannot be defined as a single expression. The absolute value function is a good example of this. Recall that $f(x) = |x|$ is defined by two equations: $f(x) = x$ if $x \geq 0$ and $f(x) = -x$ if $x < 0$. These two 'pieces' can be written as follows

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases} \longrightarrow$$

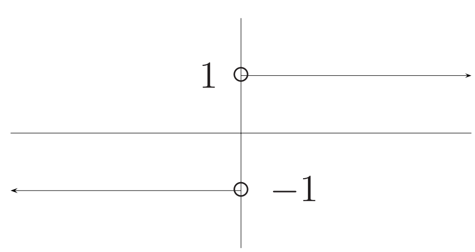


Exercise 1: Write a piecewise definition for $f(x) = |x - 3|$. Sketch the graph of f.
Answer

Here's a more complicated absolute value function.

Example: Define and sketch $g(x) = \dfrac{|x|}{x}$.

Definition: $g(x) = \begin{cases} \dfrac{x}{x} = 1 & \text{if } x > 0 \\ \dfrac{-x}{x} = -1 & \text{if } x < 0. \end{cases}$ Note: $g(0)$ is undefined.



II. Piecewise Functions

Let's analyze the piecewise function defined by

$$f(x) = \begin{cases} -x+1, & x \le -1 \\ 2, & -1 < x < 3 \\ x^2 - 4, & x \ge 3. \end{cases}$$

To help with input, think of $f(x)$ as follows:

$$f(x) = \begin{cases} 1^{\text{st}} \text{ piece}, & x \le -1 \\ 2^{\text{nd}} \text{ piece}, & -1 < x < 3 \\ 3^{\text{rd}} \text{ piece}, & x \ge 3. \end{cases}$$

In general, first look at the input intervals to select the appropriate piece to use for output.

1) Evaluate the following: $f(-3)$, $f(-1)$, $f(e)$, $f(3)$.

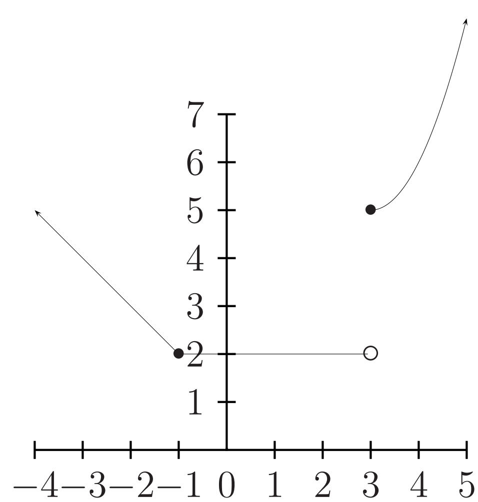
Ans.

$$\left.\begin{matrix} f(-3) = 4 \\ f(-1) = 2 \end{matrix}\right\} \text{from } 1^{\text{st}} \text{ piece}$$

$f(e) = 2$ from 2^{nd} piece, $e \approx 2.7$

$f(3) = 5$ from 3^{rd} piece

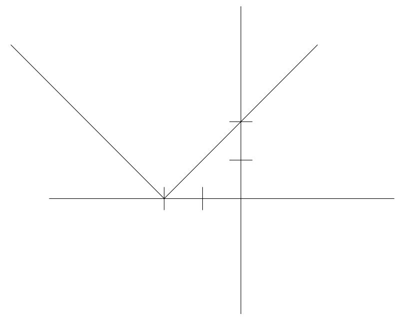
2) Here is the graph of f. Pay particular attention to the endpoints of the input intervals. Notice how this graph still passes the Vertical Line Test.



PRACTICE PROBLEMS for Topic 2 – Piecewise-Defined Functions

2.1. Define each absolute value function in piecewise form. Sketch a graph.

Ex. $f(x) = |x+2|$ Ans. $f(x) = \begin{cases} x+2, & x \geq -2 \\ -(x+2), & x < -2 \end{cases}$



a) $f(x) = |x-1|$ b) $f(x) = |2x+3|$ Answers

2.2. Let a function be 'defined' as follows

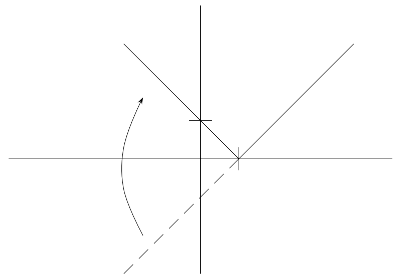
$$f(x) = \begin{cases} -x^2 - 1, & x \leq 0 \\ 2, & 0 < x < 4 \\ \sqrt{x}, & x \geq 4. \end{cases}$$

a) Find $f(-2)$, $f(0)$, $f(\pi)$, $f(x^2+5)$.

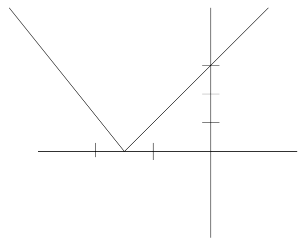
b) Sketch a graph of f. Answers

ANSWERS to PRACTICE PROBLEMS (Topic 2 – Piecewise-Defined Functions)

2.1. a) $f(x) = \begin{cases} x-1, & x \geq 1 \\ -(x-1), & x < 1 \end{cases}$



2.1. b) $f(x) = \begin{cases} 2x+3, & x \geq -3/2 \\ -(2x+3), & x < -3/2 \end{cases}$



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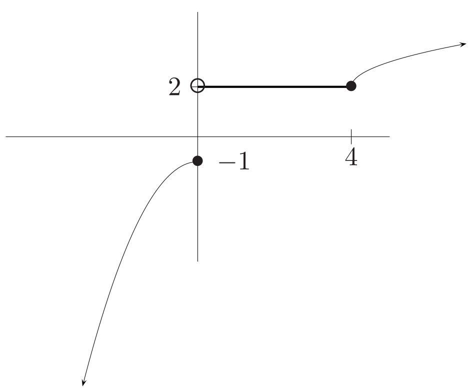
2.2. a)

$$\begin{aligned} f(-2) &= -5 \\ f(0) &= -1 \\ f(\pi) &= \ \ 2 && \text{because } \pi \approx 3.14 \\ f(x^2+5) &= \sqrt{x^2+5} && \text{because } x^2+5 > 4 \text{ for all } x. \end{aligned}$$

b)



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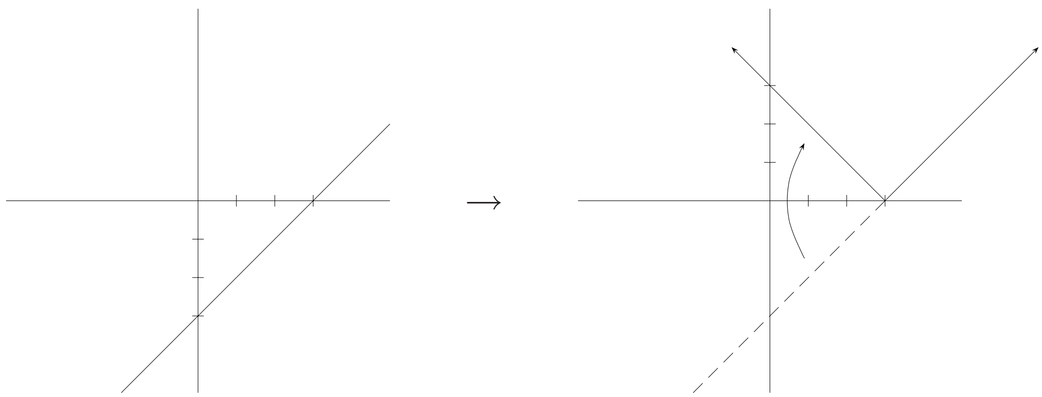
Beginning of Topic 150 Review Topics Skills Assessment

Write a piecewise definition for $f(x) = |x - 3|$. Sketch the graph of f.

Answers:

$$f(x) = |x - 3| = \begin{cases} x - 3 & \text{if } x \geq 3 \\ -(x - 3) & \text{if } x < 3 \end{cases}$$

$f(x) = x - 3$ $\qquad$ $f(x) = |x - 3|$



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