MATH 150 – TOPIC 4

SIMPLIFICATION–FUNCTION BEHAVIOR

I. In calculus, you will have to simplify expressions called derivatives (if $f(x)$ is a function, $f'(x)$ denotes its derivative). Even when fractional and negative exponents are present, factoring and cancelling are done in the same manner that would be used if the exponents were positive integers. Below are several examples of this type of simplification.

Example: Factor: $4x^{1/3}(x-1)+3x^{4/3}$.

Solution: $x^{1/3}[4(x-1)+3x] = x^{1/3}(7x-4)$.

Example: Express the following as a single quotient with only positive exponents.

$$(x^2-3)^{1/2}+\frac{1}{2}x(x^2-3)^{-1/2}(2x).$$

Solution:

$$\begin{aligned}&(x^2-3)^{-1/2}[(x^2-3)+x^2] \qquad \text{factor out } (x^2-3)^{-1/2}\\ &= \frac{2x^2-3}{(x^2-3)^{1/2}}\end{aligned}$$

* An alternative method would be to write the expression in the form $(x^2-3)^{1/2}+\dfrac{x^2}{(x^2-3)^{1/2}}$ and "Add".

Example: Express in simplest form: $\dfrac{(x^2+1)^2\cdot 3-3x(2)(x^2+1)}{(x^2+1)^4}$.

Solution:

$$\begin{aligned}&\frac{3(x^2+1)[(x^2+1)-2x]}{(x^2+1)^4}\\ &= \frac{3(x^2-2x+1)}{(x^2+1)^3}\\ &= \frac{3(x-1)^2}{(x^2+1)^3}.\end{aligned}$$

II. Now we'll look at how simplifying a function is necessary in order to investigate the behavior of that function (this is helpful when graphing a function).

Example: Given $f(x) = \dfrac{3x^2}{2\sqrt{x+1}} + 2x\sqrt{x+1}$;

a) simplify f;

b) determine its domain;

c) find intervals where $f > 0$ and $f < 0$.

Solution: a)
$$\begin{aligned} f(x) &= \frac{3x^2}{2\sqrt{x+1}} + 2x\sqrt{x+1} \\ &= \frac{3x^2}{2\sqrt{x+1}} + \frac{4x(x+1)}{2\sqrt{x+1}} \\ &= \frac{7x^2+4x}{2\sqrt{x+1}} \\ &= \frac{x(7x+4)}{2\sqrt{x+1}}. \end{aligned}$$

b) Domain: Since the denominator must remain real and non-zero, $D : (-1, -\infty)$.

c) Sign behavior is best determined by a table of signs. Critical values occur whenever $f(x) = 0$ or whenever $f(x)$ is undefined.

Critical values: 0, $-4/7$, -1

	-1 to $-4/7$	$-4/7$ to 0	0 onward
$x(7x+4)$	$+$	$-$	$+$
$2\sqrt{x+1}$	$+$	$+$	$+$
$f(x)$	$+$	$-$	$+$

Conclusions: $f(x) = 0$ at $x = -4/7;\ 0$ (x intercepts or zeroes)

$f(x) > 0$ on $(-1, -4/7)$ and $(0, \infty)$
$f(x) < 0$ on $(-4/7, 0)$.

PRACTICE PROBLEMS for Topic 4 – Simplification–Function Behavior

4.1. a) Factor out $x^{1/2}$ from $3x^{1/2} + 2x^{3/2}$.

b) Factor out $x^{-1/2}$ from $3x^{1/2} - x^{-1/2}$.

c) Factor out $(x^2+2)^{-2}$ from $x(x^2+2)^{-1} + 3(x^2+2)^{-2}$.

4.2. Simiplify each expression.

a) $2(x+1)^{1/3} + 3x(x+1)^{4/3}$ b) $(x+1)^{1/2}(2) + (2x+1) \cdot \dfrac{1}{2}(x+1)^{-1/2}$

4.3. Change the expression on the left into an equivalent form given at the right.

a) $x^{1/2} + (x-4) \cdot \dfrac{1}{2}x^{-1/2}$; $\dfrac{3x-4}{2\sqrt{x}}$

b) $\dfrac{-\frac{1}{2}(25-x^2)^{-1/2}(-2x)}{25-x^2}$; $\dfrac{x}{(25-x^2)^{3/2}}$

c) $\dfrac{x}{3(x-1)^{2/3}} + (x-1)^{1/3}$; $\dfrac{4x-3}{3(x-1)^{2/3}}$

4.4. Let $f(x) = \dfrac{-5x(x-4)}{2\sqrt{5-x}}$. Find the domain; determine the intervals where $f > 0$ and $f < 0$.

ANSWERS to PRACTICE PROBLEMS
(Topic 4 – Simplification–Function Behavior)

4.1 a) $x^{1/2}(3+2x)$

b) $x^{-1/2}(3x-1)$

c) $(x^2+2)^{-2}[x(x^2+2)+3] = (x^2+2)^{-2}(x^3+2x+3)$

4.2. a) $(x+1)^{1/3}[2+3x(x+1)]$

$$= (x+1)^{1/3}(3x^2+3x+2)$$

b) $(x+1)^{-1/2}\left[2(x+1)+\frac{1}{2}(2x+1)\right]$

$$= (x+1)^{-1/2}\left[\frac{4(x+1)+(2x+1)}{2}\right]$$

$$= \frac{6x+5}{2(x+1)^{1/2}}$$

4.3. a) $x^{-1/2}\left[x+\frac{1}{2}(x-4)\right]$

$$= x^{-1/2}\left[\frac{2x+(x-4)}{2}\right]$$

$$= \frac{3x-4}{2\sqrt{x}}$$

b) $\dfrac{x}{(25-x^2)(25-x^2)^{1/2}}$

$$= \frac{x}{(25-x^2)^{3/2}}$$

4.3. c) $\dfrac{x}{3(x-1)^{2/3}} + (x-1)^{1/3}$

$$= \frac{x}{3(x-1)^{2/3}} + \frac{3(x-1)}{3(x-1)^{2/3}}$$

$$= \frac{4x-3}{3(x-1)^{2/3}}$$

4.4. Domain: $(-\infty, 5)$
Critical values: 0, 4, 5

zeroes

0 4 5

	$(-\infty, 0)$	(0,4)	(4,5)
$-5x(x-4)$	$-$	$+$	$-$
$2\sqrt{5-x}$	$+$	$+$	$+$
$f(x)$	$-$	$+$	$-$

$f(x) > 0$ on $(0, 4)$

$f(x) < 0$ on $(-\infty, 0)$ and (4,5)

Beginning of Topic Skills Assessment