MATH 250 – REVIEW TOPIC 4

Simplifying Radicals

Have you had any difficulty with radical simplification? Here is a quick assessment.

Simplify:

a) $\sqrt{4x^2}$

b) $\sqrt{4+x^2}$

c) $\sqrt{4+4x+x^2}$

d) $\sqrt{4+4\tan^2\theta}$

Answers.

a) $2|x|$

b) already in simplest form

c) $\sqrt{(2+x)^2} = |2+x|$

d) $\sqrt{4(1+\tan^2\theta)} = \sqrt{4\sec^2\theta}$

$= 2|\sec\theta|$

Recall: The definition of $\sqrt{x^2} = |x|$. If we know $x \geq 0$, then $\sqrt{x^2} = x$. If we know $x < 0$, then $\sqrt{x^2} = -x$.

That's a reasonable start, but it doesn't meet CALC II demands. Here are examples of radical simplification that will be used in CALC II.

Ex. 1. $\sqrt{\frac{1}{4}x^2 - x + 1} = \sqrt{\frac{1}{4}(x^2-4x+4)} = \sqrt{\frac{1}{4}(x-2)^2} = \frac{1}{2}|x-2|$

Ex. 2. $\sqrt{1+\frac{4x^2}{(1-x^2)^2}} = \sqrt{\frac{(1-x^2)^2+4x^2}{(1-x^2)^2}} = \sqrt{\frac{1+2x^2+x^4}{(1-x^2)^2}}$

$= \sqrt{\frac{(1+x^2)^2}{(1-x^2)^2}} = \frac{1+x^2}{|1-x^2|}$

Ex. 3.
$$\sqrt{1+\left(2x-\frac{1}{8x}\right)^2} = \sqrt{1+\left(\frac{16x^2-1}{8x}\right)^2}$$
$$= \sqrt{\frac{64x^2+256x^4-32x^2+1}{64x^2}}$$
$$= \sqrt{\frac{(16x^2+1)^2}{64x^2}}$$
$$= \frac{16x^2+1}{|8x|}$$

Ex. 4. $\sqrt{9-4x^2}$ if $x = \dfrac{3}{2}\sin\theta$;

$$\sqrt{9-4\left(\frac{9}{4}\sin^2\theta\right)} = \sqrt{9-9\sin^2\theta} = \sqrt{9(1-\sin^2\theta)} = \sqrt{9\cos^2\theta}$$
$$= 3|\cos\theta|$$

Ex. 5. $\sqrt{1+[f'(x)]^2}$ if $f(x) = \dfrac{2}{3}(x^2-1)^{3/2}$.

First $f'(x) = 2x(x^2-1)^{1/2}$, then

$$\sqrt{1+[f'(x)]^2} = \sqrt{1+4x^2(x^2-1)}$$
$$= \sqrt{4x^4-4x^2+1}$$
$$= \sqrt{(2x^2-1)^2}$$
$$= |2x^2-1|$$

All of the radical simplification covered in our examples will be used extensively in evaluating integrals.

Practice Problems

"Practice doesn't always make perfect, but it sure helps"

Simplify the following.

4.1 $\sqrt{3x^2 - 6x + 3}$ Answer

4.2 $\sqrt{9\sec^2\theta - 9}$ Answer

4.3 $\sqrt{1 - 4x^2}$ if $x = \dfrac{1}{2}\cos\theta$ Answer

4.4 $\sqrt{e^{2x} - 4}$ if $e^x = 2\sec\theta$ Answer

4.5 $\sqrt{1 + \left(x - \dfrac{1}{4x}\right)^2}$ Answer

4.6 $\sqrt{1 + [f'(y)]^2}$ if $x = \dfrac{y^3}{3} + \dfrac{1}{4y}$ Answer

Answers to Practice Problems

4.1 $\sqrt{3(x-1)^2} = \sqrt{3}\,|x-1|$

Return to Problem

4.2 $\sqrt{9(\sec^2\theta - 1)} = \sqrt{9\tan^2\theta} = 3|\tan\theta|$

Return to Problem

4.3 $\sqrt{1 - 4\left(\frac{1}{4}\cos^2\theta\right)} = \sqrt{\sin^2\theta} = |\sin\theta|$

Note: A trig substitution is necessary to simplify the radical.

Return to Problem

4.4 $\sqrt{4\sec^2\theta - 4} = \sqrt{4(\sec^2\theta - 1)} = 2|\tan\theta|$

Return to Problem

4.5 $\sqrt{1 + \left(\frac{4x^2-1}{4x}\right)^2} = \sqrt{\frac{16x^2 + 16x^4 - 8x^2 + 1}{16x^2}} = \sqrt{\frac{(4x^2+1)^2}{16x^2}} = \frac{4x^2+1}{|4x|}$

Return to Problem

4.6 $f'(y) = y^2 - \frac{1}{4y^2}$;

$$\sqrt{1 + \left(\frac{4y^4-1}{4y^2}\right)^2} = \sqrt{\frac{16y^4 + 16y^8 - 8y^4 + 1}{16y^2}} = \sqrt{\frac{(4y^4+1)^2}{16y^2}} = \frac{4y^4+1}{|4y|}$$

Return to Problem