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Nabendu Pal

Department of Mathematics,
University of Louisiana at Lafayette

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ANALYSIS OF NON-NEGATIVE OBSERVATIONS SUBJECT TO TWO FACTORS USING GAMMA MODELS

[ABSTRACT]

A two-factor analysis of variance (ANOVA) is widely used in design of experiments when experimental units are subjected to two factors (i.e., potential sources of variations). However, such an analysis, which uses the F-tests, is dependent on three critical assumptions, - (i) all the main and interaction effects as well as the unexplained error term are additive to explain the response variable; (ii) the errors are all independent and follow a normal distribution; and (iii) the error variances, though unknown, are all equal (i.e., the errors are homoscedastic). In many engineering and biological studies, where the observations are non-negative to begin with, it is often found that one or more of the above assumptions is/are not tenable. Further, the observations tend to exhibit positively skewed distributions as seen from sample histograms. In such situations, the standard operating procedure (SOP) of the two-factor ANOVA calls for a suitable (Box-Cox type) transformation, so that the transformed observations can follow the aforementioned model assumptions. There are two practical difficulties faced by the researchers with the transformed observations: (a) the transformed observations lose their relevance to the original problem, and the resultant unit(s) of the transformed observations can be meaningless, and (b) it becomes a subjective call to come up with the most appropriate transformation of the data, i.e., one transformation can make the data adhere to one assumption while another transformation can make the data follow another assumption closely. faced with such a dilemma we offer a completely new paradigm where the non-negative observations, influenced by two factors, are modeled by gamma distributions with unknown shape and scale parameters which are dependent on the corresponding factor levels. We then proceed with testing the main effects (whether the main effects of a factor are all equal or not) and interaction effects (whether the interactions exist or not). To test a null hypothesis against a suitable alternative, we first derive the likelihood ratio test (LRT) based on its asymptotic Chi-square distribution. But since the asymptotic LRT (henceforth called 'ALRT') may not work well for small to moderate sample sizes we then propose a parametric bootstrap (PB) test based on the LRT statistic which does not use the Chi-square distribution, rather finds its critical value automatically through simulation. The PB test using LRT statistic (henceforth called 'PBLRT') appears to work very well in terms of maintaining the nominal level as seen from our comprehensive simulation study. Further, we present some real-life datasets to buttress the applicability of our proposed PBLRT over the classical ALRT, and to show how the inferences may differ from the ones based on traditional ANOVA.

SIU CARBONDALE
SOUTHERN ILLINOIS UNIVERSITY
DEPARTMENT OF MATHEMATICS