[20] 1. Compute the following limits. If the limit does not exist, explain why. Do not use L'Hopital's rule.

a) $\lim_{x\to\infty} \frac{3+4x^2}{6-x-x^2}$

$$= \lim_{x\to\infty} \frac{\frac{3}{x^2} + \frac{4x^2}{x^2}}{\frac{6}{x^2} - \frac{x}{x^2} - \frac{x^2}{x^2}} = \lim_{x\to\infty} \frac{\cancel{\frac{3}{x^2}}^{0} + 4}{\cancel{\frac{6}{x^2}}^{0} - \cancel{\frac{x}{x}}^{0} - 1} = \frac{4}{-1} = -4$$

b) $\lim_{x\to 2} \frac{x^2+2x-8}{x^2-8x+12}$ $= \frac{4+4-8}{4-16+12} = \frac{0}{0}$

$$\lim_{x\to 2} \frac{(x+4)\cancel{(x-2)}}{\cancel{(x-2)}(x-6)} = \frac{2+4}{2-6} = \frac{6}{-4} = -3/2$$

c) $\lim_{x\to 0} \frac{2x}{\tan 3x}$ $= \frac{2(0)}{\tan(3\cdot 0)} = \frac{0}{0}$

$$\lim_{x\to 0} \frac{2x}{\frac{\sin 3x}{\cos 3x}} = \lim_{x\to 0} \frac{2x\cos 3x}{\sin 3x} = \lim_{x\to 0} \frac{2\cancel{x}(\cos(3x))}{(3\cancel{x})\left(\frac{\sin 3x}{3x}\right)} \to 1$$

$$= \frac{2\cos(0)}{3} = \frac{2(1)}{3}$$

d) $\lim_{x\to -1} \frac{x+2}{(x+1)^2}$ $= \frac{-1+2}{(-1+1)^2} = \frac{1}{0^2}$ DNE

$$\lim_{x\to -1^+} \frac{x+2}{(x+1)^2} = +\infty$$

$$\lim_{x\to -1^-} \frac{x+2}{(x+2)^2} = +\infty$$

So $\boxed{\infty}$ Check if lim from Left = lim from right

$= \boxed{\frac{2}{3}}$

$-(x^2+2xh+h^2+2)$

[10] 2. Suppose $f(x) = \dfrac{1}{x^2+2}$. Find $f'(x)$ from the DEFINITION of the derivative.

$$\lim_{h\to 0} \frac{\frac{1}{(x+h)^2+2} - \frac{1}{x^2+2}}{h} = \lim_{h\to 0} \frac{\frac{x^2+2}{((x+h)^2+2)(x^2+2)} - \frac{\boxed{(x+h)^2+2}}{(x^2+2)((x+h)^2+2)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\cancel{x^2}+\cancel{2}-\cancel{x^2}-2xh-h^2-\cancel{2}}{((x+h)^2+2)(x^2+2)}}{h} = \lim_{h\to 0} \frac{\cancel{h}(-2x-h)}{(x^2+2)((x+h)^2+2)} \cdot \frac{1}{\cancel{h}}$$

$$= \frac{-2x-0}{(x^2+2)(x^2+2)}$$

$$= \boxed{\frac{-2x}{(x^2+2)^2}}$$

[36] 3. Find $f'(x)$ for the following functions. You need not simplify your answers.

a) $f(x) = 2\tan 3x - 4e^{2x}$

$f' = 2 \cdot \sec^2(3x) \cdot 3 - 4e^{2x} \cdot 2$

b) $f(x) = \sin^2 x + \sin x^2$ $= (\sin x)^2 + \sin(x^2)$

$f' = 2\sin x \cos x + \cos(x^2) \cdot 2x$

c) $f(x) = \dfrac{x^2 + 3x}{4 + \cos 2x}$

$$f' = \frac{(2x+3)(4+\cos 2x) - (-\sin(2x))\cdot 2(x^2+3x)}{(4+\cos(2x))^2}$$

d) $\cancel{f(x)}\ y = (1 + \sin x)^{2x}$

$\ln y = \ln(1+\sin x)^{2x}$

$\ln y = 2x\ln(1+\sin x)$

$\frac{1}{y}\cdot y' = 2\ln(1+\sin x) + \frac{1}{1+\sin x}\cdot(\cos x\cdot 2x)$

$y' = \left(2\ln(1+\sin x) + \frac{2x\cos x}{1+\sin x}\right)(1+\sin x)^{2x}$

e) $f(x) = \displaystyle\int_3^{x^2} \frac{dt}{1+t^3}$

$$\frac{d}{dx}\int_3^{x^2}\frac{dt}{1+t^3} = \frac{1}{1+(x^2)^3}\cdot 2x = \boxed{\frac{2x}{1+x^6}}$$

f) $f(x) = x^2 \ln(1 + \sec 3x)$

$$f' = 2x\ln(1+\sec 3x) + \frac{1}{1+\sec 3x}\cdot\sec(3x)\tan(3x)\cdot 3\cdot x^2$$

[8] 4. Find dy/dx by implicit differentiation if $ye^x - xe^y = x^2$.

$$\frac{dy}{dx}e^x + e^x y - \left(1 \cdot e^y + e^y \frac{dy}{dx}(x)\right) = 2x$$

$$\frac{dy}{dx}e^x + e^x y - e^y - xe^y\frac{dy}{dx} = 2x$$

$$\underbrace{\frac{dy}{dx}e^x - xe^y\frac{dy}{dx}}_{\text{factor out a } dy/dx} = 2x - e^x y + e^y$$

$$\boxed{\frac{dy}{dx} = \frac{2x - e^x y + e^y}{e^x - xe^y}}$$

[10] 5. Find an equation of *the* tangent line to $f(x) = x \ln x$ at $x = 2$.

$$f' = 1 \cdot \ln x + \frac{1}{\cancel{x}} \cdot \cancel{x}$$

$$m = 1 \cdot \ln 2 + 1$$

$$x = 2$$
$$y = 2\ln 2$$

$$\boxed{y - 2\ln 2 = (\ln 2 + 1)(x - 2)}$$

[8] 6. Find the absolute maximum and absolute minimum of $f(x) = \dfrac{x^2 - 4}{x^2 + 4}$ on $[-4, 4]$.

$$f' = \frac{2x(x^2+4) - 2x(x^2-4)}{(x^2+4)^2} = \frac{\cancel{2x^3} + 8x - \cancel{2x^3} + 8x}{(x^2+4)^2} = \frac{16x}{(x^2+4)^2}$$

$$\frac{16x}{(x^2+4)^2} = 0 \qquad 16x = 0 \qquad x = 0$$

undef. nowhere since $x^2 + 4 \neq 0$.

$$f(-4) = \frac{16-4}{16+4} = \frac{12}{20} = \frac{3}{5}$$

$$f(0) = \frac{-4}{4} = -1$$

$$f(4) = \frac{12}{20} = \frac{3}{5}$$

abs Max is $\frac{3}{5}$, abs. Minis -1

$d = 2r$ $r = d/2$

[10] 7. If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm.

$\frac{dSA}{dt} = -1 \frac{cm^2}{min}$

$\frac{dd}{dt} = ?$ when $d = 10$

$SA = 4\pi r^2 = 4\pi\left(\frac{d}{2}\right)^2 = 4\pi\frac{d^2}{4} = \pi d^2$

$\underbrace{\frac{dSA}{dt}}_{-1} = 2\pi \underbrace{d}_{10} \cdot \frac{dd}{dt}$

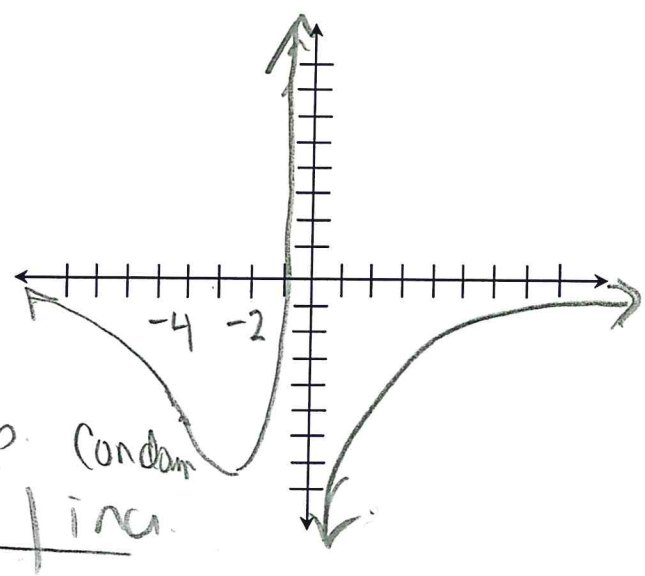
$-1 = 2\pi(10)\frac{dd}{dt} \Rightarrow \boxed{\frac{dd}{dt} = \frac{-1}{20\pi} \frac{cm}{min}}$

[8] 8. Graph the function which satisfies the following conditions.

$\lim_{x\to\infty} f(x) = 0$, $\lim_{x\to-\infty} f(x) = 0$, $\lim_{x\to 0^-} f(x) = \infty$ $\lim_{x\to 0^+} f(x) = -\infty$

~~$f' > 0$ on $(-2, 0) \cup (0, \infty)$;~~ ~~$f' < 0$ on $(-\infty, -2)$~~

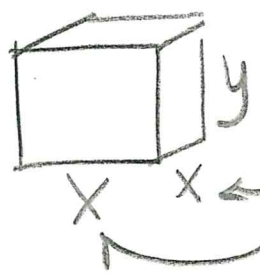
$f'' > 0$ on $(-4, 0)$; $f'' < 0$ on $(-\infty, -4) \cup (0, \infty)$



condon. decr	conup decr	conup inc	condon inc

-4 2 0

[10] 9. A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimizes the amount of material used.



$M = x^2 + xy + xy + xy + xy$

Bottom, L&R, front & Back

$M = x^2 + 4xy$ subject to $x^2y = 32000$

$y = \frac{32000}{x^2}$

$M = x^2 + 4x\left(\frac{32000}{x^2}\right) = x^2 + \frac{128000}{x} = x^2 + 128000x^{-1}$

$M' = 2x - 128000x^{-2} = 2x - \frac{128000}{x^2} = 0$

$\Rightarrow 2x^3 - 128000 = 0$

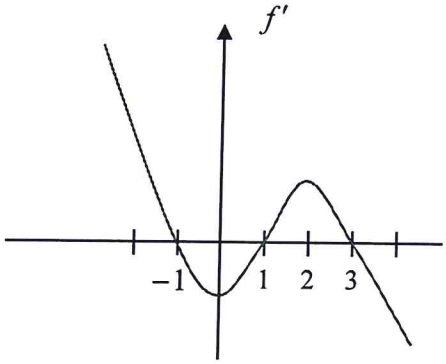
$x^3 = 64000$

$x = 40$

$y = \frac{32000}{40^2} = 20$

$\boxed{\begin{array}{l} 40\text{cm} \\ \times 40\text{cm} \\ \times 20\text{cm} \end{array}}$

[10] 10. Suppose that the DERIVATIVE f' of a function f has the graph



This graph is not the graph of the function. It is the graph of the derivative of f.

a) Find the intervals where f is increasing/decreasing.

inc $(-\infty,-1) \cup (1,3)$

decr $(-1,1) \cup (3,\infty)$

b) Find the intervals where f is concave up/down. → where f' is decreasing

Con up $(0,2)$ Con down $(-\infty,0) \cup (2,\infty)$

[30] 11. Evaluate the following indefinite integrals.

a) $\int (x+2)(x+3)dx$ $= \int x^2+5x+6\, dx$

$= \boxed{\frac{x^3}{3} + \frac{5x^2}{2} + 6x + C}$

b) $\int (\tan x - 3\csc^2 x)dx$

$\int \tan x = \int \frac{\sin x}{\cos x} dy$

$u = \cos x$

$du = -\sin x\, dy$

$= \boxed{-\ln|\cos x| + 3\cot x + C}$

$-\int \frac{1}{u} du$

$= -\ln|u| + C$

$= -\ln|\cos x| + C$

c) $\int \frac{dx}{x \ln x}$

$u = \ln x$

$du = \frac{1}{x} dy$

$= \int \frac{1}{x} \cdot \frac{1}{\ln x} dx$

$= \int \frac{1}{u} du = \ln|u| + C$

$= \boxed{\ln|\ln x| + C}$

d) $\int xe^{-x^2} dx$

$u = -x^2$

$du = -2x\, dx$

$\frac{du}{-2} = x\, dx$

$= \int e^u \frac{du}{-2} = -\frac{1}{2} e^u + C$

$= \boxed{-\frac{1}{2} e^{-x^2} + C}$

$\int \frac{x}{x^3} + \frac{x^{1/2}}{x^3}$ $\quad \frac{1}{2} - 3 \cdot \frac{2}{2}$

e) $\int \frac{x+\sqrt{x}}{x^3}\,dx = \int x^{-2} + x^{-5/2}\,dx$

$= \frac{x^{-1}}{-1} + \frac{x^{-3/2}}{-3/2} = \boxed{\frac{-1}{x} - \frac{2}{3}x^{-3/2} + C}$

[18] 12. Find the following definite integrals.

a) $\int_0^3 x\sqrt{x+1}\,dx$

$x(x+1)^{1/2}$

$u = x+1 \Rightarrow x = u-1$

$du = dx$

$\int_1^4 (u-1)u^{1/2}\,du$

u-bounds

$x=0 \Rightarrow u=1$

$x=3 \Rightarrow u=4$

$= \int_1^4 u^{3/2} - u^{1/2}\,du = \frac{2u^{5/2}}{5} - \frac{2u^{3/2}}{3}\Big|_1^4 = \frac{2}{5}(4)^{5/2} - \frac{2}{3}(4)^{3/2} - \left(\frac{2}{5}(1)^{5/2} - \frac{2}{3}(1)^{3/2}\right)$

$= \frac{2}{5}(32) - \frac{2}{3}(8) - \left(\frac{2}{5} - \frac{2}{3}\right)$

$= \frac{64}{5} - \frac{16}{3} - \frac{2}{5} + \frac{2}{3}$

$= \frac{62}{5} - \frac{14}{3}$

$= \frac{186 - 70}{15} = \boxed{\frac{116}{15}}$

$(0,1)$

b) $\int_0^{\pi/8} (\sin 4x + 3\cos 2x)\,dx$

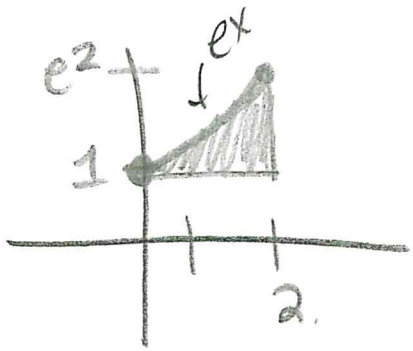
$= -\frac{\cos(4x)}{4} + \frac{3\sin(2x)}{2}\Big|_0^{\pi/8}$

$= -\frac{\cos \pi/2}{4} + \frac{3\sin \pi/4}{2} - \left(-\frac{\cos(0)}{4} + \frac{3\sin(0)}{2}\right)$

$= 0 + \frac{3}{2}\cdot\frac{\sqrt{2}}{2} - \left(-\frac{1}{4}\right) = \boxed{\frac{3\sqrt{2}}{4} + \frac{1}{4}}$

[20] 13. Let R be the region bounded by $y = e^x$ and $y = 1$ and $x = 2$. Find the following.

a) Area of R.



$y = 1$

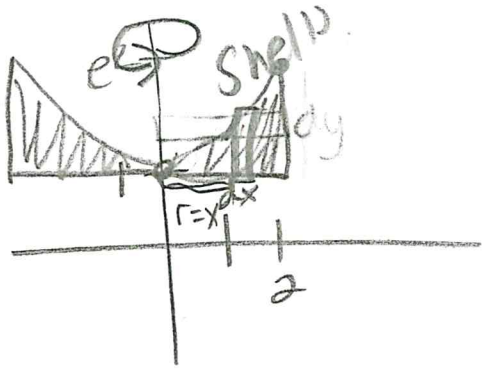
$1 = e^x$

$x = 0, \; x = 2$

$$\int_0^2 e^x - 1 \, dx = e^x - x \Big|_0^2$$

$$= e^2 - 2 - (e^0 - 0) = e^2 - 2 - 1$$

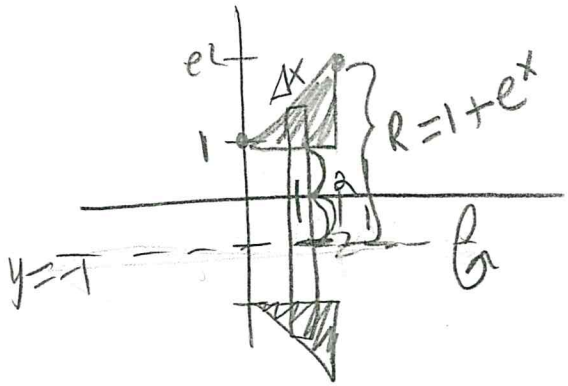
b) Volume of the solid obtained by rotating R about the line $y-$axis. SETUP ONLY. $= \boxed{e^2 - 3}$



$$\int_0^2 2\pi x (e^x - 1) \, dx$$

$2\pi r h$

c) Volume of the solid obtained by rotating R about the line $y = -1$. SETUP ONLY.



$$\int_0^2 \pi (1 + e^x)^2 - \pi (2)^2 \, dx$$

$$\pi R^2 - \pi r^2$$

$R \to 1 + e^x$, $r \to 2$