Part I. There are 8 problems in Part I, each worth 5 points. No partial credit will be given, so be careful. Circle the correct answer.

1) Determine an equivalent expression for $\sin(\theta - \frac{\pi}{2})$.

a) $\sin\theta$　　b) $\cos\theta$　　c) $-\sin\theta$　　d) $-\cos\theta$　　e) Not a, b, c, or d

$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

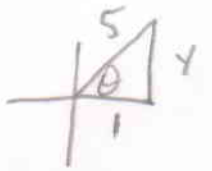
$\sin(\theta - \frac{\pi}{2}) = \sin(\theta)\cos(\frac{\pi}{2}) - \cos(\theta)\sin(\frac{\pi}{2})$

$= \sin(\theta)\cdot 0 - \cos(\theta)(1)$

$= -\cos(\theta)$

2) If θ is an acute angle and $\cos\theta = \frac{1}{5}$, what is $\sin\theta$?

a) $\frac{24}{5}$　　b) $\frac{4}{5}$　　c) $\frac{24}{25}$　　d) $\frac{\sqrt{24}}{5}$　　e) Not a, b, c, or d



$1^2 + y^2 = 5^2$

$y^2 = 24$

$y = \sqrt{24}$

$\sin\theta = \frac{\sqrt{24}}{5}$

3) Find the dot product of u and v if $\vec{u} = <2, -3>$ and $\vec{v} = <1, -2>$.

a) 6
b) -1
c) <3, 3>
d) 8
e) Not a, b, c, or d

$u \cdot v = u_1 v_1 + u_2 v_2$

$= 2 \cdot 1 + -3 \cdot -2$

$= 2 + 6$

$= 8$

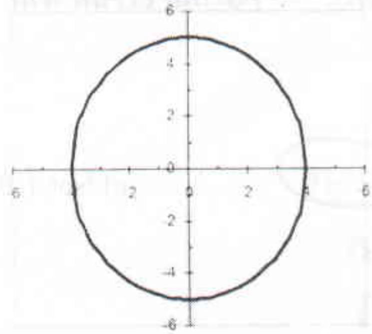
4) Find the supplement of $\frac{2\pi}{5}$.

a) $\frac{3\pi}{5}$　　b) $\frac{\pi}{10}$　　c) $\frac{-2\pi}{5}$　　d) $\frac{\pi}{5}$　　e) Not a, b, c, or d

$\pi - \frac{2\pi}{5}$

$\frac{5\pi}{5} - \frac{2\pi}{5} = \frac{3\pi}{5}$

5) What is the equation for the following graph?



ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$

a) $\frac{x^2}{5} + \frac{y^2}{4} = 1$ b) $\frac{x^2}{25} + \frac{y^2}{16} = 1$ c) $\frac{x^2}{4} + \frac{y^2}{5} = 1$ d) $\frac{x^2}{16} + \frac{y^2}{25} = 1$ e) Not a, b, c, or d

6) Which of the following are coterminal to an angle of 70 degrees?

a) 20 degrees b) -290 degrees c) -430 degrees d) 290 degrees e) Not a, b, c or d

70 ± 360

$\begin{array}{r} 360 \\ -70 \\ \hline 290 \end{array}$

7) Find the directrix of the parabola $y^2 = -8(x+1)$.

a) y= -1 b) x = 1 c) y = 1 d) y = -2 e) Not a, b, c, or d

$(-1, 0)$

$-8 = 4p$

$p = -2$

$\frac{C}{B} = \frac{+3}{2}$

8) What is the phase shift for the graph: $y = -3\sin(2x - 3) + 5$?

a) 5 b) 3/2 c) 3 d) -3/2 e) not a,b,c or d

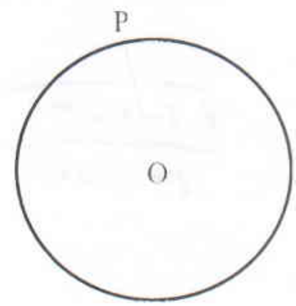
Part II. Partial credit will be given here. Show all your work. Each problem is worth 6 points.

9) Given vectors $u = \langle -2, 4 \rangle$ and $v = \langle -1, -1 \rangle$ what is $< 3u - 2v >$?

$\langle -6, 12 \rangle - \langle -2, -2 \rangle$

$\langle -4, 14 \rangle$

10) Suppose that P is a point on a circle with a radius of 6 inches and the ray OP is rotating with angular speed 60 degrees per second. (Round to nearest tenth.)



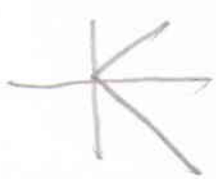
a) Find the speed in radians per second.

$\left(\frac{60^\circ}{1\text{ sec}}\right)\left(\frac{1\pi\text{ rad}}{180^\circ}\right) = \frac{1.0\pi\text{ rad}}{3\text{ sec}}$

b) Find the distance travelled by P along the arc after 5 seconds. (i.e. Arc length.)

$\left(\frac{1.0\pi\text{ rad}}{3\text{ s}}\right)(5\text{ s})(6) = 10\pi''$

11) Given $\cos\theta = .22$, state the solution set on $[0,360°)$. Approximate to nearest tenth of a degree. Show all work clearly.

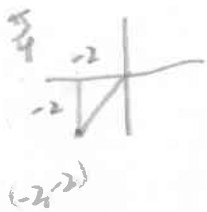


$\arccos(.22) = 77.3^\circ$

$= 282.7^\circ$

12) If the rectangular coordinates of a point are $(-2, -2)$, what are its polar coordinates (r, θ) given the following?

a) $r > 0,\ 0 \le \theta < 2\pi$ b) $r < 0,\ 0 \le \theta < 2\pi$



$2^2 + 2^2 = r^2$

$8 = r^2$

$\sqrt{8} = r$

$2\sqrt{2} =$

$\left(2\sqrt{2}, \frac{5\pi}{4}\right)$

$\left(-2\sqrt{2}, \frac{\pi}{4}\right)$

13) Rationalize the denominator .

$\frac{\sqrt{1-\sin(x)}}{\sqrt{1+\sin(x)}} \cdot \frac{\sqrt{1+\sin(x)}}{\sqrt{1+\sin(x)}} =$

$\frac{\sqrt{1-\sin^2(x)}}{1+\sin(x)} = \frac{\sqrt{\cos^2 x}}{1+\sin(x)} = \frac{\cos(x)}{1+\sin(x)}$

14) Perform the indicated operation:

a) Factor: $2sin^2x - 3cosxsinx - 2cos^2x$ b) Simplify: $\frac{cos^2x+sin^2x}{1-sin^2x} = \frac{1}{\cos^2 x} = \sec^2 x$

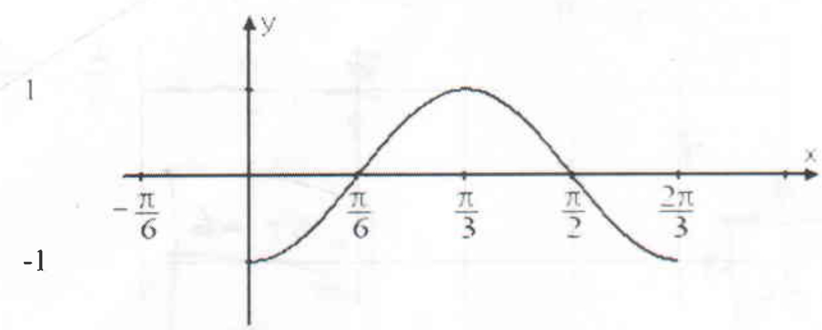
$(2\sin x + \cos(x))(\sin x - 2\cos x)$

Part III. Partial credit will be given here. Show all your work. Each problem is worth 12 points.

15) Write an equation for each.

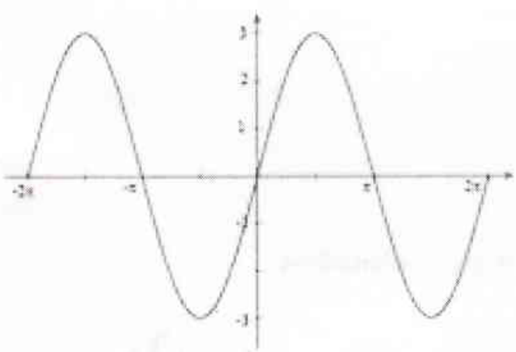
period $= \frac{2\pi}{3} = \frac{2\pi}{B}$

$B = 3$



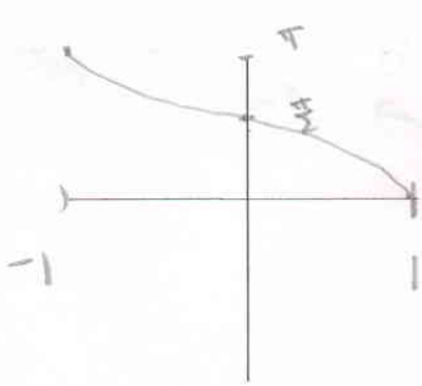
a) Answer $-\cos(3x)$

period $= 2\pi = \frac{2\pi}{B}$ $B = 1$



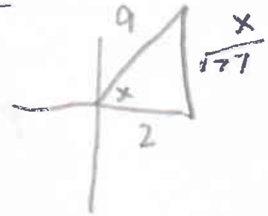
b) Answer $3\sin(x)$

c) Graph $f(x) = cos^{-1}x$.
Label axes with at least 2 ticks each.



16) Given that $\cos(x) = \frac{2}{9}$ and x is a quadrant I angle, $\cos(y) = \frac{4}{5}$ where y is in quadrant IV angle. Give exact values!!! You should not need a calculator.

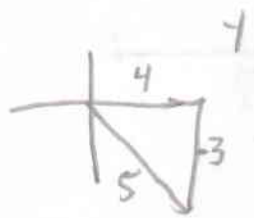
a) Find sin(2y).



$2\sin y \cos y$

$2 \cdot \left(\frac{-3}{5}\right)\left(\frac{4}{5}\right)$

$= \boxed{-\frac{24}{25}}$



$81 - 4 =$

b) Find $\sin(x + y)$.

$\sin x \cos y + \cos x \sin y$

$\frac{\sqrt{77}}{9} \cdot \frac{4}{5} + \frac{2}{9} \cdot \frac{-3}{5}$

$= \boxed{\frac{4\sqrt{77} - 6}{45}}$

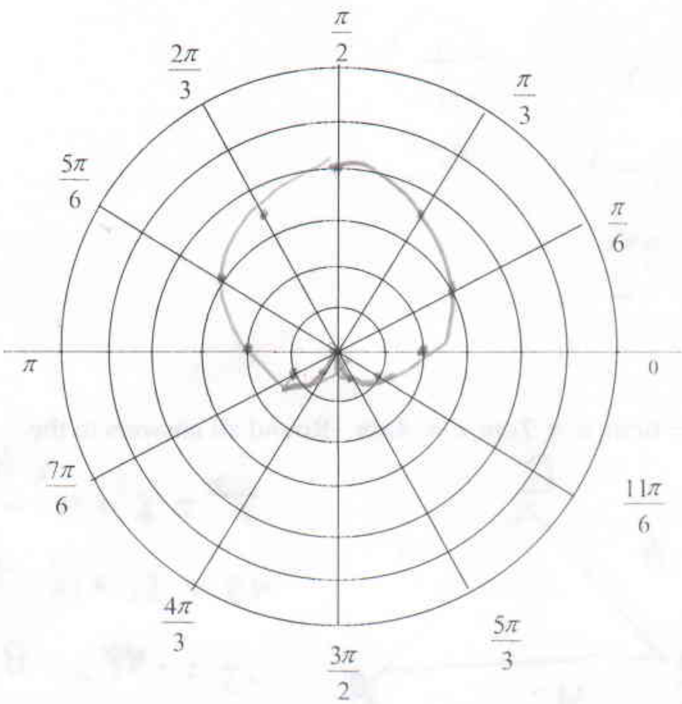
17) Verify (prove): $cot^2(x) = \frac{\csc^2(x)}{1+tan^2x}$. Include all steps and explanations.

$\frac{\csc^2 x}{1 + \tan^2 x} = \frac{\csc^2 x}{\sec^2 x}$ → pog identity

$\frac{\csc^2 x}{\sec^2 x} = \frac{\cos^2 x}{\sin^2 x}$ → def of csc and sec

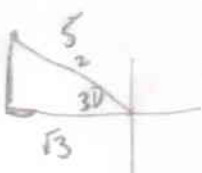
$\frac{\cos^2 x}{\sin^2 x} = \cot^2 x$ → def of cot

18) a) Graph the polar equation $r = 2\sin(\theta)+2$ on the axes below. (7 pts)



θ	r
0	2
$\frac{\pi}{6}$	3
$\frac{\pi}{3}$	3.732
$\frac{\pi}{2}$	4
$\frac{2\pi}{3}$	3.732
$\frac{5\pi}{6}$	3
π	2

b) Convert the following polar form into rectangular form: $(5, 150^0)$ (5 pts)



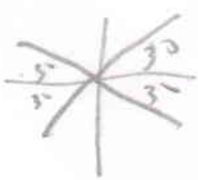
$r = 5$ $\cos(150) = \frac{x}{5}$ $\sin(150) = \frac{y}{5}$

$\left(-5\sqrt{3}, \frac{5}{2}\right)$

19) Solve the following.

a) Find **all** solutions to $4\cos^2 x - 3 = 0$. (6 pts) Express in terms of degrees.

Note: **All solutions** are the same as **general solutions.**



$4\cos^2 x = 3$

$\cos^2 x = \frac{3}{4}$

$\cos x = \frac{\pm\sqrt{3}}{2}$

$30^\circ, 150^\circ, 210^\circ, 330^\circ$

$30^\circ + 360n$
$150^\circ + 360n$
$210^\circ + 360n$
$330^\circ + 360n$

n is an integer

b) $2\sin^2 x = 1 - \sin x$ on $[0, 2\pi)$ (6 pts)

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

0, -1

$2\sin x = 1$ or $\sin x = -1$

$\sin x = \frac{1}{2}$ $\quad x = \frac{3\pi}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

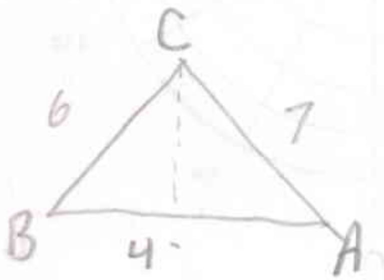
20) Solve the triangle ABC with sides $a = 6cm, b = 7cm, c = 4cm$. Round all answers to the nearest tenth (one decimal place).

A= 58.8°

B= 86.4°

C= 34.8°

180°



$7^2 = 6^2 + 4^2 - 2(6)(4)\cos B$

$49 = 36 + 16 - 48\cos B$

$-3 = -48\cos B$

$16 = 49 + 36 - 2(7)(6)\cos C$

$6^2 = 7^2 + 4^2 - 2(7)(4)\cos A$ $\quad B = 86.4$

-15

$c =$

$A = 58.8^\circ$

21) Change: $4(x-3)^2 + 8y^2 = 40$ into standard form. Then graph. Label vertices or center as well as any foci.

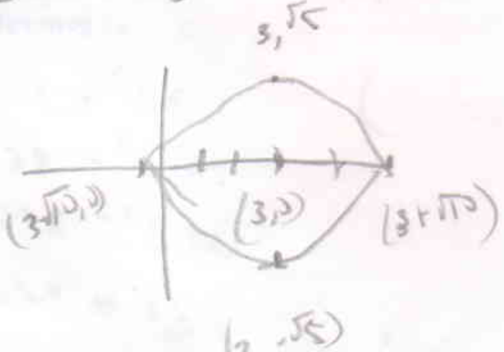
$\frac{4(x-3)^2}{40} + \frac{8y^2}{40} = 1$

vertices = $(3, 0)$

$a = \sqrt{10}$

$b = \sqrt{5}$

$\frac{(x-3)^2}{10} + \frac{y^2}{5} = 1$



PART IV. Here are 6 problems. Do any 4, but only 4. Each is worth 10 points. Be sure to check the box for each problem to be graded.

☐ Grade

22) Find cube roots of $2-2i$. Leave answers in trig form. (Exact answers in degrees.)

$(2-2i)^{\frac{1}{3}}$ $\quad 360 \times \frac{1}{3} = 120°$

$\frac{1}{3}$

$[2\sqrt{2}\ \cos 315 + i\sin 315]$

$(2\sqrt{2})^{\frac{1}{3}}[\cos(315 \cdot \frac{1}{3}) + i\sin 315 \cdot \frac{1}{3}]$

$z_1 = (2\sqrt{2})^{\frac{1}{3}}[\cos 105° + i\sin 105]$

$z_2 = (2\sqrt{2})^{\frac{1}{3}}[\cos 225° + i\sin 225°]$

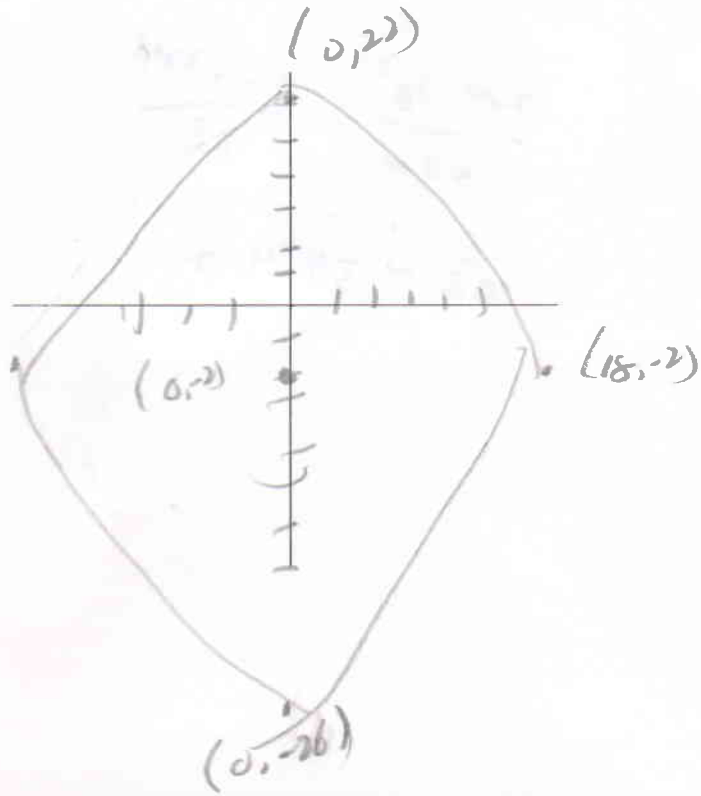
$z_3 = (2\sqrt{2})^{\frac{1}{3}}[\cos 345° + i\sin 345°]$

☐ Grade

23) Graph the following. Indicate and label all critical information.

$$\frac{(y-2)^2}{16 \cdot 36} + \frac{x^2}{9 \cdot 36} = 36/36 \quad \frac{(y-2)^2}{576} + \frac{x^2}{324} = 1$$

- Center: $(0,-2)$
- Vertices: $(0,22)$ $(0,-26)$
- Foci: $\sqrt{252}$ (15.87)



$a = 18$

$b = 24$

$(-18,-2)$

$c^2 = |18^2 - 24^2|$

(REMDINDER: Do 4 of the 6 problems in this section and check the box next to the ones I should grade!)

☐ Grade

24) Write the trigonometric expression as an algebraic expression in terms of $u(u > 0)\csc(\tan^{-1} u)$.

$\frac{1}{\sin}(\tan^{-1} u)$

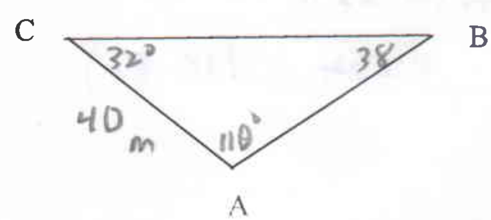
$\sin\theta = \frac{u}{\sqrt{u^2+1}}$

$\csc\theta = \frac{\sqrt{u^2+1}}{u}$

☐ Grade

25) Points A & B are on opposite sides of a lunar crater. Point C is 40 m from point A. The measure of angle BAC is 110 degrees and the measure of angle ABC is 38 degrees. What is the width of the crater (distance from point A to B)?

$\frac{\sin 38^\circ}{40\text{ m}} = \frac{\sin(32^\circ)}{\overline{AB}}$



$\overline{AB} = 34.4$ m

(REMINDER: Do 4 of the 6 problems in this section and check the box next to the ones I should grade!)

☐ Grade

26) Prove the following identity. $\frac{\tan x - \cot x}{\sin x \cos x} = \sec^2 x - \csc^2 x$

$\frac{\tan x}{\sin x \cos\alpha} - \frac{\cot x}{\sin\alpha \cos\alpha} \rightarrow$ subtraction

$\frac{\frac{\sin x}{\cos x}}{\sin\alpha \cos x} - \frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha \cos\alpha} \rightarrow$ def of trig functions

$\rightarrow$ cancel terms

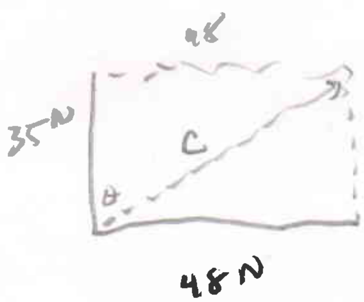
$\frac{1}{\cos^2\alpha} - \frac{1}{\sin^2\alpha} \rightarrow$ def of trig functions

$\sec^2\alpha - \csc^2\alpha$

☐ Grade

27) Two forces of 48 N and 35 N act on objects at right angles. (Round to nearest tenth and use degrees.)

a) Find the magnitude of the resultant vector.



$c^2 = 48^2 + 35^2$

$c = 59.4$ N

b) Find the angle the resultant vector makes with the smaller force.

$\tan(\theta) = \frac{48}{35}$

$\theta = 53.9°$