(9 points)

1. Evaluate each of the following. **Show all work and give a single integer as your answer**.

   a) $P(5,5)$

   b) $P(10,4)$

   d) $C(12,5)$

In problems 2, 3 and 4 you may leave your answers in terms of products, powers, permutations and/or combinations. You need not simplify your answers.

(10 points)

2. Serial numbers for a product are to be made using 2 letters followed by 3 numbers. If the letters are to be taken from the first 10 letters of the alphabet with no repeats and the numbers are to be taken from the 10 digits (0-9) with no repeats.

   How many serial numbers are possible?

(15 points)

3. A salesperson contacts prospective customers by telephone, and estimates that 45% of all telephone calls results in a sale. The salesperson makes 10 telephone calls. Find the probability that

(a) Exactly 8 telephone calls result in a sale.

(b) No telephone calls result in a sale.

(c) At least 2 of the 10 telephone calls result in a sale.

(15 points)

4. Suppose that 8 female and 5 male applicants have been successfully screened for 5 positions. If the 5 positions are filled at random from the 13 finalists, what is the probability of selecting

a) 3 females and 2 males?

b) 5 females?

c) At least 3 males?

(10 points)

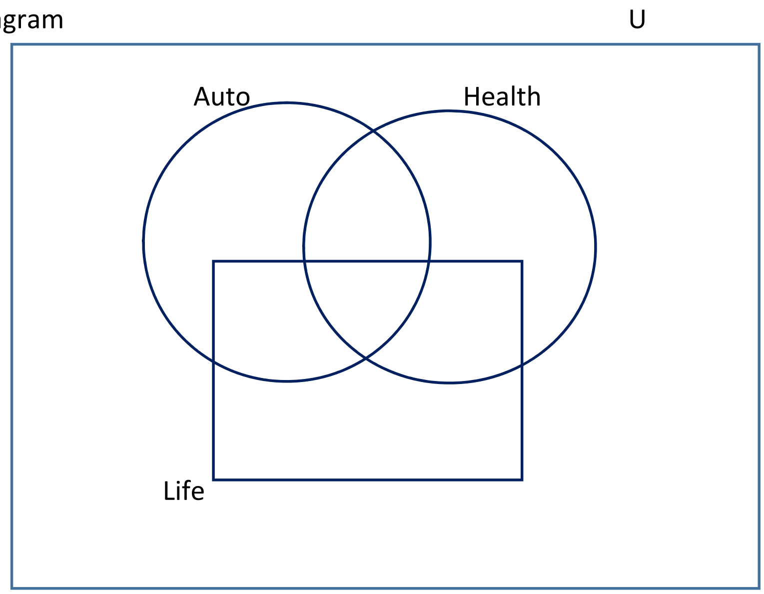
5. a) Find the equation of the line through the points (-2, 2) and (-8, 14). Give your answer in slope-intercept form.

b) Are the lines with equations $6x+3y=4$ and $y=-2x+7$ parallel, intersecting or coincident? Justify your answer.

(12 points)

6. An insurance agent sells auto insurance, health insurance, and life insurance. She checks sales at the end of the first year and finds that she has sold 95 auto, 70 health, and 50 life insurance policies. However, a number of customers purchased more than one type of insurance policy: 20 bought both auto and health, 15 both health and life, 10 both auto and life, and 7 bought all three types. .

a) Complete the Venn diagram



b) Find the total number of customers.

c) How many customers bought exactly 2 types of insurance?

d) How many bought only health insurance?

(9 points)

7. How many different permutations are there of all of the letters in 'STATISTICS'? Show all work and give a single integer as your answer.

(15 points)

8. A company produces 1000 refrigerators a week at three plants. Plant A produces 250 refrigerators a week, plant B produces 400 refrigerators a week and plant C produces 350 refrigerators a week. Production records indicate that 3% of the refrigerators produced at plant A will be defective, 5% of those produced at plant B will be defective, and 7% of those produced at plant C will be defective. All the refrigerators are shipped to a central warehouse.

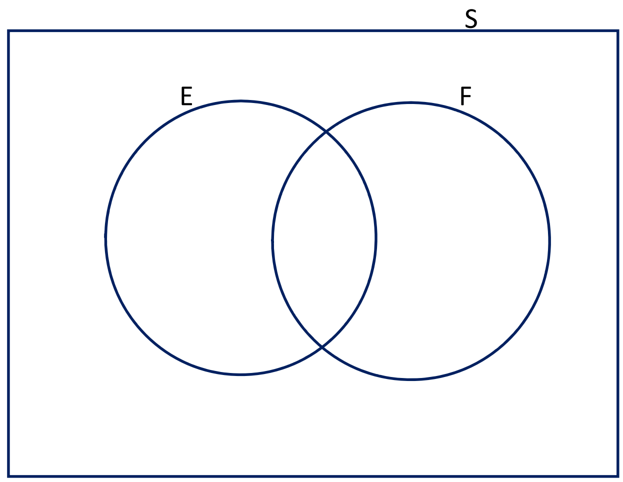
a) Construct the tree diagram for this problem, complete with the branches labeled and the assigned probabilities.

b) If a refrigerator at the warehouse is found to be defective, what is the probability that it was produced at plant A?

(18 points)

9. Let E and F be events of sample space S. The P (E ∪ F) = 0.7, P(E) = 0 .6, P (F) = 0.3

a) Complete the Venn diagram



b) Find P (E ∩ F).

c) Find P (E ∪ $\bar{F}$).

d) Find P (E| F) .

e) Are E and F independent ? Justify your answer.

(12 points)

10. Given the three matrices $A = \begin{bmatrix} 0 & 1 & 3 \\ 2 & -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 4 \\ 1 & 0 & 0 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$. Find

a) $B - 2I_3$

b) AB (if possible)

c) BA (if possible)

d) C^{-1} (the inverse of C)

(15 points)

11. Solve the following system by using **reduced row-echelon** form.

$$
\begin{aligned}
2x + 3y + z &= 11 \\
x + 2z &= 10 \\
3x + y + 4z &= 23
\end{aligned}
$$

(9 points)

12. The following augmented matrices represent systems of equations in terms of x, y, and z. For each find the general solution or state that no solution exists.

a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$ b) $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 7 \end{array}\right]$ c) $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(12 points)

13. Formulate an LP model for the following problem. DO NOT ATTEMPT TO SOLVE IT.

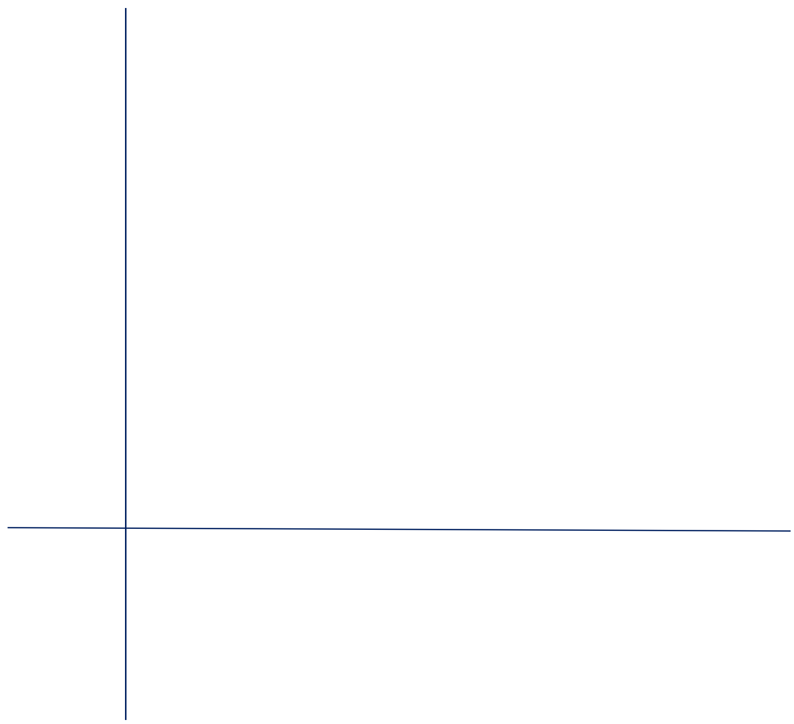
A toy manufacturer makes two different kinds of model cars: the Porsche and the Ferrari. They are made of steel and aluminum. Each Porsche requires 1 unit of steel and 2 units of aluminum, each Ferrari requires 3 units of steel and 4 units of aluminum. The company has 5,500 units of steel and 10,000 units of aluminum available. The number of Porsches made must be at least twice of the number of Ferraris made. The total number of model cars made must be at least 1,000. If $10 profit is made on each Porsche and $15 on each Ferrari, how many of each model car should the toy manufacturer make in order to maximize its profit?

(12 points)

14. Solve the following linear programming problem:

Maximize $P = 10x + 20y$ subject to the following constraints:
$x \geq 0$, $y \geq 0$, $6x + 2y \geq 36$, $2x + 4y \geq 32$, $y \leq 20$ $x \leq 16$.

a) Sketch the feasible region



b) List the corner points of the feasible region

c) Report the complete solution.

(12 points)

15. For each tableau perform one of the following steps:

    1) If a pivot is required, write 'pivot needed', circle the pivot element, but do not pivot.
    2) If there is no optimal solution, say so and state why.
    3) If the problem is finished, report the complete solution.

a)

$$\begin{array}{c} \text{BV} \\ x_1 \\ x_2 \\ P \end{array} \begin{array}{c} \begin{array}{ccccccc} \text{P} & \text{x}_1 & \text{x}_2 & \text{s}_1 & \text{s}_2 & \text{s}_3 & \text{RHS} \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 2 & 1 & 1 & 8 \\ 0 & 0 & 1 & 2 & 2 & 4 & 12 \\ 1 & 0 & 0 & 4 & 0 & 5 & 25 \end{pmatrix} \end{array}$$

b)

$$\begin{array}{c} \text{BV} \\ s_1 \\ s_2 \\ s_3 \\ P \end{array} \begin{array}{c} \begin{array}{cccccccc} \text{P} & \text{x}_1 & \text{x}_2 & \text{x}_3 & \text{s}_1 & \text{s}_2 & \text{s}_3 & \text{RHS} \end{array} \\ \begin{pmatrix} 0 & 0 & 4 & 2 & 1 & 0 & 0 & 12 \\ 0 & 1 & 5 & 2 & 0 & 1 & 0 & 25 \\ 0 & 0 & 0 & -2 & 0 & 0 & 1 & 9 \\ 1 & 2 & -3 & -1 & 1 & 3 & 3 & 18 \end{pmatrix} \end{array}$$

c)

$$\begin{array}{c} \text{BV} \\ s_1 \\ s_2 \\ s_3 \\ P \end{array} \begin{array}{c} \begin{array}{cccccccc} \text{P} & \text{x}_1 & \text{x}_2 & \text{x}_3 & \text{s}_1 & \text{s}_2 & \text{s}_3 & \text{RHS} \end{array} \\ \begin{pmatrix} 0 & 1 & -2 & 5 & 1 & 0 & 0 & 15 \\ 0 & 3 & 0 & 2 & 0 & 1 & 0 & 11 \\ 0 & 0 & -3 & -1 & 0 & 0 & 1 & 22 \\ 1 & 3 & -4 & 2 & 0 & 0 & 0 & 45 \end{pmatrix} \end{array}$$

(15 points)

16. Use the simplex method to solve the following LP problem.

Maximize $P = 3X_1 + X_2$ subject to the following constraints:

$X_1 \geq 0$, $X_2 \geq 0$, $X_1 + 3X_2 \leq 5$, $X_1 - X_2 \leq 1$