

Math 108 Skills Assessment

The purpose of this test is purely diagnostic (before beginning your review, it will be helpful to assess both strengths and weaknesses). All of the test problems are essential to Math 108. Answers are provided, and each answer has references to the relevant review topics. If anything is unclear, the review material should help.

You may click on the blue words if you wish to jump to an answer or the review topics.

If you would like to print the Skills Assessment so you can work it out on paper, please click [Print](#).

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Pre-Assessment: Basic Computation

Put that calculator away, grab a pencil and some scratch paper and try doing some math like your teachers (and parents) used to do it.

SHORTCUTS ENCOURAGED

1. $-3 + 4(5 - 2)^{-2}$

2. $\frac{1/3 - 3/4}{6}$

3. $-5(20/21)(-7/15)$

4. $\left(\frac{2}{3}\sqrt{6}\right)^2$

5. $(9/4)^{-3/2}$

6. $5\sqrt{3} - 2\sqrt{12}$

7. $\frac{4.8 \times 10^6}{6 \times 10^9}$

8. $\frac{2^{14}}{8^5}$

9. $\log_3 81$

10. $\log_9 \frac{1}{3}$

11. $\log .00001$

12. $-|3 - 4(2)|$

[Answer](#)

Algebra Assessment

1. **Properties of Real Numbers.** For any real numbers a and b , indicate TRUE or FALSE. If False, give an example to support your answer.

a) $|a + b| = |a| + |b|$ b) $\sqrt{a^2 + b^2} = a + b$ c) $\sqrt{a^2} = a$
 d) $\frac{a - b}{b - a} = -1, \quad a \neq b$

Answer

2. **Laws of Exponents.** Simplify. (Final answers should include only positive exponents.)

a) $\frac{(3x^2y^3)^2}{6x^5y}$ b) $\frac{3^{-1}x^2}{-4x^{-4}}$ c) $(x^2y^{1/3})^{1/2}(x^{-3/4}y^{2/3})$

Answer

3. **Polynomials.** Perform the indicated operation.

a) $(2x^2 - 5y)^3$ b) $(x + 4)(2x - 1) - (2x - 3)^2$ c) $\frac{3x^3 - x^2 + x - 2}{x^2 + 2}$
 divided by $x^2 + 2$

Answer

4. **Factoring.** Factor completely.

a) $x^2y^2 - 7xy - 30$ b) $4x^3 + 14x^2 - 30x$
 c) $x^3 - 125$ d) $a^2 - 2ab + 3ac - 6bc$

Answer

5. **Rational Expressions.** Perform the indicated operation.

a) Simplify: $\frac{2x^2 - 6x}{2x^2 - 5x - 3}$ b) $\frac{5a^2 + 12a + 4}{a^4 - 16} \div \frac{25a^2 + 20a + 4}{a^2 - 2a}$
 c) Find the least common denominator for $\frac{1}{x^3}$, $\frac{x}{(x - 2)^2}$, and $\frac{2x + 3}{2x - x^2}$.
 d) $4 + \frac{3}{\mu + 5} - \frac{2}{\mu}$ e) $\frac{3}{1 + \frac{1}{n}}$

Answer

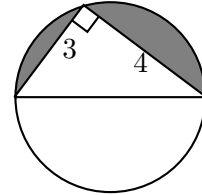
6. **Simplifying Roots.**

a) $\sqrt[3]{16a^4b^6}$ b) $(2\sqrt{6} - \sqrt{3})^2$ c) $\frac{\sqrt{a} - \sqrt{b}}{a - b}$; rationalize the numerator.

Answer

7. Geometry Topics.

- a) Find the perimeter of a right triangle whose hypotenuse is 4 and one leg measures 2.
- b) A rectangular pool (20 ft by 30 ft) is surrounded by a cement walk 4 ft in width.
- Find the area of the walk.
 - If the cement for the walk were poured to a depth of 6 inches, how many cubic feet of cement would be needed?
- c) When a right triangle is inscribed in a circle, the hypotenuse is also a diameter. If the triangle's legs are 3 and 4,
- find the area of the circle;
 - find the shaded area.

[Answers](#)**8. Linear Equations.**

- a) Solve for x : $(5x - 7)(2x + 1) - 10x(x - 4) = 0$
- b) Solve for x : $\frac{3}{7x - 2} = \frac{9}{3x + 1}$
- c) The formula for calculating the surface area of a rectangular box is: $s = 2(lw + wh + lh)$. Solve for h .

[Answers](#)**9. Word Problems.**

- a) Six hundred people attended the afternoon showing of a movie. Adult tickets cost \$5 and students were admitted for \$2. If receipts totaled \$2400, how many students were in attendance?
- b) Sterling silver is a copper-silver alloy that is 7.5% copper by weight. How many grams of pure copper and how many grams of sterling silver should be used to prepare 200 grams of an alloy that is 10% copper by weight?
- c) At 5 a.m. a snowplow, traveling at constant speed, begins clearing a highway. At 7 a.m. a car travels the highway at a speed of 30 mph, catching up to the plow in 30 minutes. Find the speed of the plow.
- d) A single garden hose can fill a swimming pool in 8 hours. A larger hose used alone can fill the pool in 5 hours. How long would it take to fill the pool if both hoses were used simultaneously?

[Answers](#)

10. **Quadratic Equations.** Solve for x .

a) $2x^2 + 7x - 15 = 0$

b) $x(x - 5) = 6$

c) $(x - 3)^2 = 17$

d) $\frac{5x}{x-3} + \frac{4}{x+3} = \frac{90}{x^2-9}$

[Answers](#)

e) Solve by completing the square: $x^2 - 4x - 21 = 0$

[Answers](#)11. **Irrational Equations.** Solve for x .

a) $\sqrt{7-5x} = 8$

b) $\sqrt[5]{2x^2+1} - 2 = 0$

c) $\sqrt{3\sqrt{x}} = \sqrt{x}$

[Answers](#)12. **Inequalities; Interval Notation.**a) For $a > b > 0$, indicate True or False for the following statements. If false, give an example.

i) $b - a < 0$

ii) $\frac{1}{a} < \frac{1}{b}$

iii) $-a < -b$

iv) $b < b^2$

[Answers](#)

b) Write the interval form for each:

i) $x \leq 3$

ii) $-2 < x < 4$

iii) $x < -2$ or $x \geq 5$

c) Solve each inequality. Draw a graph of the solution.

i) $-3 < 5 - 2x < 7$

ii) $(x - 4)^2 > x(x + 12)$

[Answers](#)13. **Absolute Value Equations.**

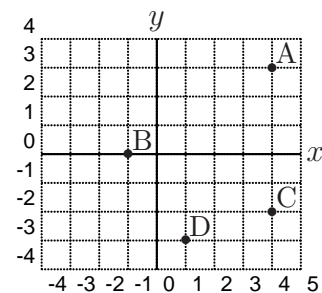
a) $|x + 3| = 5$

b) $|3 - x| = -1$

c) $|3x - 2| + 3 = 7$

[Answers](#)14. **Rectangular Coordinate System.**

- A
- 1) What are the coordinates of B ? of D ?
 - 2) If a point (x, y) lies in quadrant II, $(-x, y)$ lies in _____.
 - 3) What is the slope of line segment AB ? of BD ?
 - 4) What is the slope of a line perpendicular to AC ?

[Answers](#)

14. **B** Consider the following equations

$$y = 4 - 3x, \quad 2x - 3y = 5.$$

Graph each line, labeling all intercepts.

[Answers](#)

- C** State the equation of the line in point-slope and slope-intercept forms,

- 1) with x -intercept 2 and slope of $\frac{1}{3}$;
- 2) passing through $(-1, 3)$ and parallel to $x - 2y = 4$.

[Answers](#)

15. **Graphs of Basic Functions.** On the same axis, sketch a graph of each.

- a) $y_1 = 2x$, $y_2 = -2x + 1$
- b) $y_1 = x^2$, $y_2 = x^2 - 2$
- c) $y_1 = \sqrt{x}$, $y_2 = \sqrt{-x}$, $y_3 = -\sqrt{x}$
- d) $y_1 = |x|$, $y_2 = |x + 1|$

[Answers](#)

16. **Functions.** Consider $f(x) = 3 - x^2$, $g(x) = \sqrt{x + 2}$.

- a) Find $f(-1)$, $f(1/2)$, $g(-2)$, $2g(7)$.
- b) Find all x such that $f(x) = -1$.
- c) Find all intercepts of f .
- d) Find $f(g(3))$.

[Answers](#)

*** When you have completed the assessment [click here](#).

ANSWERS – Pre-Assessment: Basic Computation

1. $-3 + \frac{4}{9} = -\frac{23}{9}$

2. $-\frac{5}{12} \cdot \frac{1}{6} = -\frac{5}{72}$

3. $= \frac{20}{9}$ Did you cancel?

4. $\left(\frac{4}{9} \cdot 6\right) = \frac{8}{3}$

5. $\left(\frac{4}{9}\right)^{3/2} = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

6. $5\sqrt{3} - 4\sqrt{3} = \sqrt{3}$

7. $.8 \times 10^{-3} = 8.0 \times 10^{-4}$

8. $\frac{2^{14}}{(2^3)^5} = \frac{2^{14}}{2^{15}} = \frac{1}{2}$

I call that a shortcut.

9. 4; because $3^4 = 81$

10. $-\frac{1}{2}$; because $9^{-1/2} = \frac{1}{3}$

11. -5; because $10^{-5} = \frac{1}{100,000} = .00001$

12. $-|-5| = -5$

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ANSWERS - Algebra Assessment

1. Properties of Real Numbers.

a) False $|-3 + 5| \neq |-3| + |5|$

b) False $\sqrt{3^2 + 4^2} \neq 3 + 4$

c) False $\sqrt{(-3)^2} \neq -3$; $\sqrt{a^2} = |a|$ is True.

d) True

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2. Laws of Exponents.

a) $\frac{9x^4y^6}{6x^5y} = \frac{3y^5}{2x}$

b) $\frac{x^6}{-4 \cdot 3} = -\frac{1}{12}x^6$

c) $(xy^{1/6})(x^{-3/4}y^{2/3}) = x^{1/4}y^{5/6}$

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3. **Polynomials.**

$$\text{a) } \underbrace{(4x^4 - 20x^2y + 25y^2)}_{(2x^2-5y)^2}(2x^2 - 5y) = 8x^6 - 60x^4y + 150x^2y^2 - 125y^3$$

$$\text{b) } 2x^2 + 7x - 4 - (4x^2 - 12x + 9) \\ - 2x^2 + 19x - 13$$

$$\text{c) } 3x - 1 - \frac{5x}{x^2 + 2}$$

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4. **Factoring.**

$$\text{a) } (xy - 10)(xy + 3)$$

$$\text{b) } 2x(2x^2 + 7x - 15) = 2x(2x - 3)(x + 5)$$

$$\text{c) } (x - 5)(x^2 + 5x + 25) \quad \text{The difference of 2 cubes.}$$

$$\text{d) } a(a - 2b) + 3c(a - 2b) = (a + 3c)(a - 2b) \quad \text{“Grouping”}$$

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5. **Rational Expressions.**

$$\text{a) } \frac{2x(x - 3)}{(2x + 1)(x - 3)} = \frac{2x}{2x + 1}$$

$$\text{b) } \frac{(5a + 2)(a + 2)}{(a^2 + 4)(a - 2)(a + 2)} \cdot \frac{a(a - 2)}{(5a + 2)^2} \quad \text{Factor and invert, then cancel.} \\ = \frac{a}{(a^2 + 4)(5a + 2)}$$

$$\text{c) } x^3(x - 2)^2$$

$$\text{d) } \frac{4\mu(\mu + 5) + 3\mu - 2(\mu + 5)}{\mu(\mu + 5)} \quad \text{LCD } \mu(\mu + 5) \\ = \frac{4\mu^2 + 21\mu - 10}{\mu(\mu + 5)} \quad \text{Since numerator is non-factorable, we can't simplify.}$$

$$\text{e) } \frac{3}{\frac{n+1}{n}} = \frac{3n}{n+1}$$

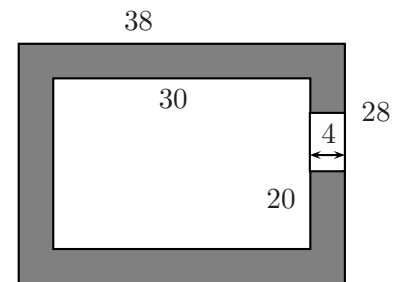
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6. **Simplifying Roots.**

- a) $\sqrt[3]{8a^3b^6(2a)} = 2ab^2\sqrt[3]{2a}$
- b) $(2\sqrt{6})^2 - 4\sqrt{18} + (\sqrt{3})^2 = 27 - 12\sqrt{2}$
- c) $\frac{\sqrt{a} - \sqrt{b}}{a - b} \cdot \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} + \sqrt{b}} = \frac{a - b}{(a - b)(\sqrt{a} + \sqrt{b})}$
 $= \frac{1}{(\sqrt{a} + \sqrt{b})}$

[Return to Problem](#)7. **Geometry Topics.**

- a) 2nd leg = $\sqrt{4^2 - 2^2} = 2\sqrt{3}$; $P = 6 + 2\sqrt{3}$
- b) i. $A = 38 \times 28 - 30 \times 20 = 464 \text{ ft}^2$
- ii. $V = \frac{1}{2}(464) = 232 \text{ ft}^3$
 To convert to $\text{yd}^3 \Rightarrow \frac{232}{27} \text{ yd}^3$
- c) i) From Pythagorean Theorem, diameter = 5.
 $A = \pi r^2 = \frac{25}{4}\pi$
- ii) Area of semicircle - Area of triangle
 $= \frac{25}{8}\pi - 6$

[Return to Problem](#)8. **Linear Equations.**

- a) $x = \frac{7}{31}$
- b) Proportions can be solved by cross multiplication.

$$3(3x + 1) = 9(7x - 2), \quad x = \frac{21}{54} = \frac{7}{18}$$

- c) After removing () and collecting like terms,

$$s - 2lw = 2wh + 2lh$$

$$\text{so } \frac{s - 2lw}{2(w + l)} = h \text{ (from factoring out } h\text{).}$$

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9. Word Problems.

a) x = number of students, $600 - x$ = number of adults.

$$2x + 5(600 - x) = 2400 \text{ yields } x = 200.$$

b)

$$\begin{array}{c} \text{“pure”} \end{array} \quad \begin{array}{c} \curvearrowright 1.0 \\ \boxed{x} \end{array} + \begin{array}{c} .075 \\ \boxed{200 - x} \end{array} = \begin{array}{c} .10 \\ \boxed{200} \end{array}$$

$$\text{so } 1(x) + .075(200 - x) = .10(200)$$

$$\text{yields } x = 5.4$$

c)

	R	\cdot	T	$=$	D
plow	R		2		$2.5R$
car	30		$1/2$		15

To overtake, distances must be equal.

$$2.5r = 15, \quad r = 6 \text{ mph}$$

d) Smaller hose fills $1/8$ pool/hr.

Larger hose fills $1/5$ pool/hr.

$$\text{working} \times \text{hours}, \quad \frac{1}{8}x + \frac{1}{5}x = 1$$

$$x = \frac{40}{13} \text{ hr.}$$

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10. Quadratic Equations.

a) $x = 3/2, -5$

b) $x^2 - 5x - 6 = 0$; $x = 6, -1$

c) By taking roots, $x = 3 \pm \sqrt{17}$. If you expand x^{-3} , you'll have to do more work and the quadratic formula to get this result.

d) $5x(x + 3) + 4(x - 3) = 90$

$$5x^2 + 19x - 102 = 0$$

$$(5x + 34)(x - 3) = 0$$

$$x = -\frac{34}{5}$$

Multiply by LCD to cancel out denominator

Put into quadratic form

Hard to find these factors, may need quad. formula

Why is $x = 3$ not a solution?

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$$\begin{aligned}
 \text{e)} \quad x^2 - 4x + 4 &= 21 + 4 \\
 (x - 2)^2 &= 25 \\
 x - 2 &= \pm 5 \\
 x &= 2 \pm 5; \quad x = 7, -3
 \end{aligned}$$

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11. Irrational Equations.

$$\text{a)} \quad x = -\frac{57}{5}$$

$$\text{b)} \quad 2x^2 + 1 = 32$$

$$x^2 = \frac{31}{2}$$

$$x = \pm \sqrt{\frac{31}{2}} \text{ or}$$

$$\begin{aligned}
 \text{c)} \quad 3\sqrt{x} &= x && \text{square both sides} \\
 9x &= x^2 && \text{square both sides} \\
 0 &= x^2 - 9x \\
 0 &= x(x - 9); \quad x = 0, 9
 \end{aligned}$$

$$\pm \frac{1}{2}\sqrt{62} \text{ if you rationalized.}$$

NOTE: Checking roots is important. Some solutions won't check and must be omitted.

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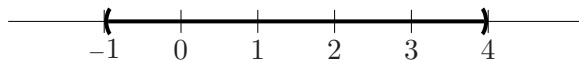
12. Inequalities; Interval Notation.

$$\text{a)} \quad \text{i) True} \qquad \text{ii) True} \qquad \text{iii) True}$$

$$\text{iv) False; } \frac{1}{2} \neq \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

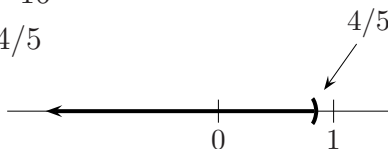
$$\text{b)} \quad \text{i) } (-\infty, 3], \qquad \text{ii) } (-2, 4), \qquad \text{iii) } (-\infty, -2) \cup [5, \infty)$$

$$\text{c)} \quad \text{i) } -8 < -2x < 2 \Rightarrow 4 > x > -1 \text{ or } -1 < x < 4$$



$$\text{ii) } -20x > -16$$

$$x < 4/5$$



Notice in both i) and ii) we reversed order. Why??

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13. **Absolute Value Equations.**

- a) $x + 3 = 5$ or $-(x + 3) = 5$; $x = 2, -8$
 b) No solution; how can the absolute value end up negative?
 c) $3x - 2 = 4$ or $-(3x - 2) = 4$; $x = 2, -2/3$

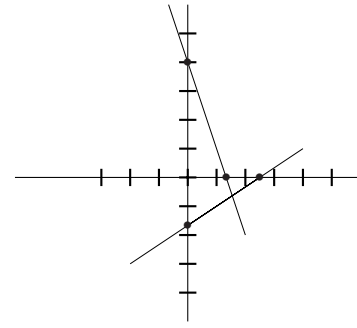
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14 **Rectangular Coordinate System.**

- A. 1) $(-1, 0), (1, -3)$
 2) I
 3) $m\overline{AB} = 3/5, m\overline{BD} = -3/2$
 4) $m\overline{AC} = \text{undefined}, \perp m = 0$

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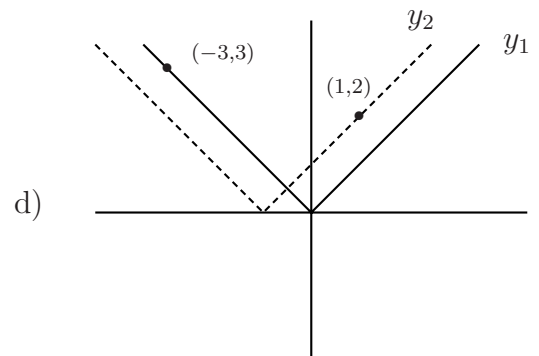
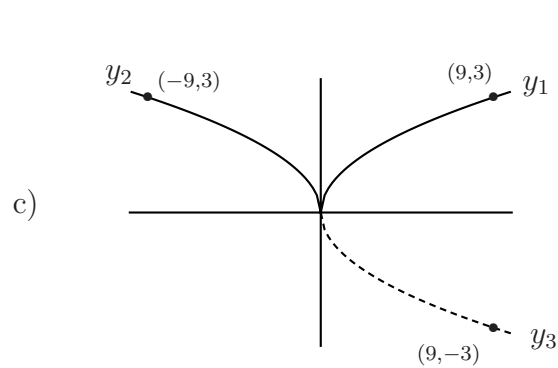
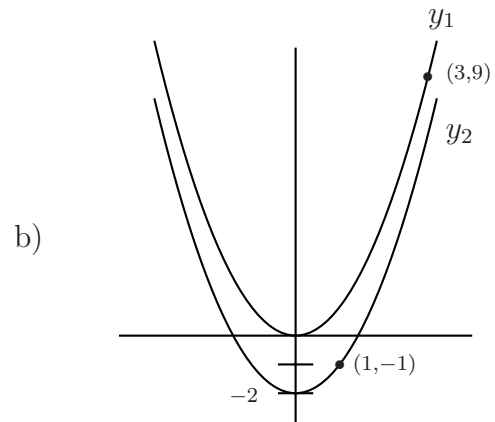
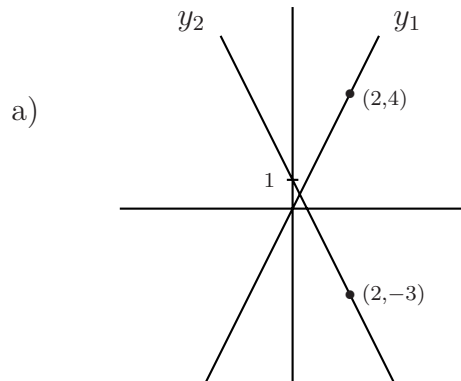
- B. 1) $y = 4 - 3x$ $(4/3, 0), (0, 4)$
 $2x - 3y = 5$ $(5/2, 0), (0, -5/3)$



- C. 1) $y = \frac{1}{3}(x - 2), y = \frac{1}{3}x - \frac{2}{3}$
 2) $m = \frac{1}{2}; y - 3 = \frac{1}{2}(x + 1), y = \frac{1}{2}x + \frac{7}{2}$

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15 Graphs of basic functions.



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16. Functions.

a) $f(-1) = 2$, $f(1/2) = 1\frac{1}{4}$, $g(-2) = 0$, $2g(7) = 2(3) = 6$

b) Solve $3 - x^2 = -1$, $x = \pm 2$

c) y int: $f(0) = 3$
 x int: $3 - x^2 = 0$, $x = \pm\sqrt{3}$

d) $f(\sqrt{5}) = 3 - (\sqrt{5})^2 = -2$

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