Abstract: Automorphic forms were first studied in the context of number theory. Among their many interesting features, they give rise to $L$-functions (functions with similar properties to the Riemann zeta function). We will discuss the use of spectral theory of automorphic forms to solve differential equations and discuss two applications of such solutions. The first application may shed light on the location of zeros of $L$-functions. The equation in question arises from a mistake that allowed for zeros of the Riemann zeta function to appear as eigenvalues for the $SL_2(\mathbb{R})$-invariant Laplace-Beltrami operator on the Poincare upper half plane and has been studied by Bombieri and Garrett. The second application provides a quantum correction in the discrepancy between general relativity and empirical data. Specifically, Green, Russo and Vanhove discovered that the low energy expansion of scattering amplitude for gravitons (hypothetical particles of gravity represented by massless string states) has coefficients which satisfy differential equations in automorphic forms. We will discuss how spectral theory of automorphic forms can be used in both of these contexts.