

1. Which of the following is not prime?

- a) 257 b) 167 c) 187 d) 241 e) All are prime.

2. Which of the following CANNOT be the sum of the digits of a square?

- a) 11 b) 4 c) 13 d) 7 e) None listed.

3. Solve for x if $\sqrt{5 + \sqrt{19 - \sqrt{x}}} = \sqrt[3]{3\sqrt{81}}$.

- a) 4 b) 9 c) 16 d) 25 e) 46

4. A box contains \$26 in nickels, dimes and quarters. If there is the same number of nickels as dimes, but twice as many quarters as nickels, how many dimes are in the box?

- a) 26 b) 30 c) 34 d) 40 e) 50

5. Two perpendicular line segments divide a large rectangle into 4 smaller rectangles. The area of three of these four small rectangles is as shown. What is the area of the other small rectangle?

6	9
8	?

- a) 12 b) 13 c) 14 d) 15 e) 16

6. Let $f(x) = \frac{3x-5}{x-2}$. Then the inverse function $f^{-1}(x)$ is
- a) $\frac{x-2}{3x-5}$ b) $\frac{3x-7}{x-5}$ c) $\frac{2x-5}{x-3}$ d) $\frac{3-5x}{x-3}$ e) $\frac{5-2x}{2-x}$
7. If $\log_{10}(2) = 0.30103$, find $\log_{10}(200)$.
- a) 3.0103 b) 0.60206 c) 2.60206 d) 2.15052 e) 2.30103
8. Evaluate the sum.
- $$\cos^2(5^\circ) + \cos^2(10^\circ) + \cos^2(15^\circ) + \cdots + \cos^2(90^\circ)$$
- a) 8 b) 9 c) 8.5 d) 9.5 e) 10
9. Suppose $x > 0$. What is the value of $\cos\{\arctan[\sin(\operatorname{arccot} x)]\}$?
- a) $\sqrt{\frac{x^2+1}{x^2+2}}$ b) $\sqrt{\frac{x^2+2}{x^2+1}}$ c) $\frac{1}{\sqrt{x^2+1}}$ d) $\frac{1}{\sqrt{x^2+2}}$ e) None listed.
10. Which of the following would, by itself, constitute evidence that $A \rightarrow B$ is false?
- a) A is true.
b) B is false.
c) A is false.
d) A is false and B is true.
e) B is false and A is true.

11. How many roots does the equation $\sqrt{x+7} + x = 13$ have?

- a) 0 b) 1 c) 2 d) 3 e) None listed.

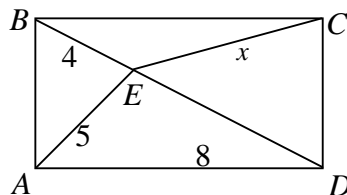
12. How many real numbers x satisfy $x^3 + x + 1 = 0$?

- a) 0 b) 1 c) 2 d) 3 e) More than 3.

13. Which of the following methods will correctly check whether 371 is a prime number?

- a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8 or 9.
b) Break 371 into 3 and 71. They are both prime so 371 must be prime.
c) Check to see whether 371 is divisible by any prime number less than 20.
d) Check to see whether 371 is divisible by any number less than 20.
e) More than one of the above.

14. Let $ABCD$ be a rectangle. Find the length of \overline{CE} .



- a) $\sqrt{60}$ b) 6.4 c) 7 d) $\sqrt{55}$ e) 7.4

15. If a regular hexagon is inscribed in a circle of radius 3, what is its area?

- a) 3 b) $\frac{9\sqrt{3}}{2}$ c) $\frac{27\sqrt{3}}{2}$ d) 27 e) None listed.

16. A die is weighted so that the '6' never occurs. The remaining numbers are equally likely. The die is thrown twice. What is the probability that the sum is 6?

- a) $\frac{6}{25}$ b) $\frac{3}{25}$ c) $\frac{1}{6}$ d) $\frac{1}{5}$ e) None.

17. The letters in **MATH FIELD DAY** are cycled as shown and placed on a number list. The next correct spelling (**MATH FIELD DAY**) occurs in a row number n . Find the value of n .

1. **MATH FIELD DAY**
2. **ATHM IELDF AYD**
3. **THMA ELDFI YDA**
- ⋮ ⋮
- n . **MATH FIELD DAY**

- a) $n = 60$ b) $n = 12$ c) $n = 32$ d) $n = 35$ e) None listed.

18. Under what conditions is the equation $(a+b)^2 = a^2 + b^2$ true?

- a) Never true. b) Always true. c) $a = 0$ d) $b = 0$ e) Either c or d.

19. Find the largest integer k such that $\frac{3}{2} \cdot \frac{2}{1} \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdots \frac{k}{k+1} \geq \frac{1}{8}$.
- a) 20 b) 21 c) 23 d) 24 e) 26
20. A circle passes through the points (0,6), (0,10) and (8,0). What is the second x -intercept?
- a) 7 b) 7.25 c) 7.5 d) 7.75 e) 9
21. For a certain value of a we have $\cos a + \sin a = \frac{5}{4}$. Find the value of expression $|\cos a - \sin a|$.
- a) $\frac{\sqrt{7}}{4}$ b) $\frac{\sqrt{5}}{4}$ c) $\frac{\sqrt{5}}{5}$ d) $\frac{1}{2}$ e) $-\frac{1}{2}$
22. John can perform a certain task in 12 days, Adam in 9 days and Bill in 18 days. In how many days will the task be finished if all three worked together? Which group of numbers contains the answer?
- a) 0, 7, 1 b) 3, 5, 8 c) 39, 40, 38 d) 4, 2, 6 e) $\frac{11}{2}, 9, 12$
23. Suppose there are m girls and n boys in a class. What is the number of ways of arranging them in a line so that all the girls are together?
- a) $m!n!$ b) $(m+1)!n!$ c) $m!(n+1)!$ d) $(m+1)!(n+1)!$ e) $m n!$

24. A father left all his money in his will to his children in the following manner:

\$1,000 to the first born and a $\frac{1}{10}$ of what remains,

\$2,000 to the second born and a $\frac{1}{10}$ of what remains,

\$3,000 to the third born and a $\frac{1}{10}$ of what remains, and so on.

When the money was distributed, each child received the same amount. How many children were there?

- a) 6 b) 7 c) 8 d) 9 e) 10

25. Five couples (husband and wife) are to be seated in a row of 11 chairs. In how many ways can they be seated if each husband is to sit next to his wife?

- a) 240 b) 2440 c) 23040 d) 11440 e) None listed.

26. What is the sum of the digits of the smallest positive integer that has a remainder of 1 when divided by 4, a remainder of 2 when divided by 5 and a remainder of 3 when divided by 6?

- a) 13 b) 12 c) 15 d) 16 e) 11

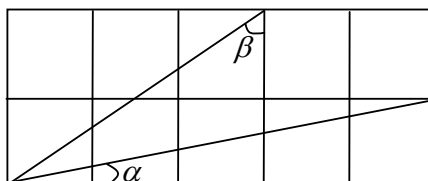
27. There are two large jars. One is filled with orange juice and another with lemonade. Exactly one cup from the orange juice jar is added to the lemonade jar. After stirring well, exactly one cup is taken from the mixed jar and put in the orange juice jar. Which of the following is a true statement?

- a) There is more orange juice in the lemonade jar than lemonade in the orange juice jar.
b) There is more lemonade in the orange juice jar than orange juice in the lemonade jar.
c) There is the same amount of the “wrong” drink in each jar.
d) It depends on the relative size of the jars and the cup.
e) None listed.

28. If $x^2 + 2x + n > 10$ for all real numbers x , then

- a) $n > -11$ b) $n < 11$ c) $n = 10$ d) $n > 11$ e) None listed.

29. Ten squares of equal size are arranged as below. Find $\beta - \alpha$.



- a) 36° b) 37.5° c) 45° d) 48° e) 52.5°

30. How many integers x in $[1, 100]$ are there such that $x^2 + x^3$ is the square of an integer?

- a) 6 b) 7 c) 8 d) 9 e) 10

31. Which of the following is a whole number?

- a) $\sec \frac{\pi}{2}$ b) $\tan^{-1} 1$ c) $\sin\left(\frac{\pi}{6}\right)$ d) $\sec\left(\frac{\pi}{3}\right)$ e) None listed.

32. Two circles intersect at A and B . A line is tangent to these circles at C and D and intersects \overline{AB} at E . If $AE = 1$ and $BE = 3$, find CD .

- a) 1 b) 2 c) 3 d) $2\sqrt{3}$ e) None listed.

33. If $\frac{y}{4} = \frac{z}{x}$, $\frac{x}{y} = \frac{3}{z}$, $\frac{z}{x} = \frac{2}{y} \Rightarrow \frac{x^2 - z^2}{y^2} = ?$

- a) 2 b) $\frac{3}{4}$ c) 1 d) $\frac{1}{2}$ e) -2

34. Let $f(x) = 2f(x+2) - x$ and $f(2) = 6$. Find $f(6)$.

- a) 2 b) 4 c) 6 d) 8 e) 10

35. How many times during one day (from midnight to midnight) does the minute and hour hands of a clock make a right angle? Find your answer in one of these groups of numbers:

- a) 1, 30 b) 12, 24 c) 46, 23 d) 21, 44 e) 22, 48

36. The U.S. Senate has 100 senators. In the U.S. Senate a committee of 5 senators is to be chosen from 22 Republicans and 18 Democratic senators. The committee is to contain at least 3 Republicans. How many committees are there?

- a) $\binom{100}{3} + \binom{100}{4} + \binom{100}{5}$
b) $\binom{100}{22} \binom{22}{3} + \binom{100}{18} \binom{18}{3}$
c) $\binom{40}{5} + \binom{40}{4} \binom{22}{1} + \binom{40}{3} \binom{22}{3}$
d) $\binom{22}{3} \binom{18}{2} + \binom{22}{4} \binom{18}{1} + \binom{22}{5}$
e) $\binom{40}{3} \binom{40}{2} + \binom{40}{4} \binom{40}{1} + \binom{40}{5}$

37. Find a solution to $(x+1)^x + (x+2)^x = (x+4)^x$.

- a) $x=0$ b) $x=1$ c) $x=2$ d) $x=5$ e) $x=\pi$

38. Suppose r_1, r_2, r_3 are the three roots to the cubic equation

$$2010 + 2011x + 2012x^2 + 2013x^3 = 0.$$

Then $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} =$

- a) $-\frac{2011}{2010}$ b) $-\frac{2012}{2011}$ c) $-\frac{2013}{2012}$ d) $-\frac{2010}{2011}$ e) $-\frac{2011}{2012}$

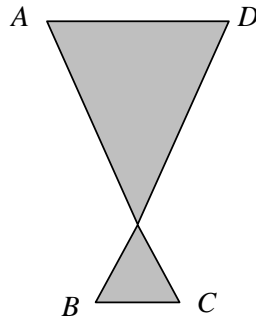
39. Find m such that $x^3 - 5x^2 + 7x + (m-5) = p(x)$ is divisible by $x-4$.

- a) -7 b) 0 c) 5 d) 7 e) 17

40. How many pairs of natural numbers $(p, p+2)$ exist, where both p and $p+2$ are prime?

- a) 0 b) 1 c) Infinitely many. d) Currently unknown. e) None listed.

41. Suppose that $\overline{AD} \parallel \overline{BC}$, $\overline{AD} = 3$, $\overline{BC} = 1$, and the distance between \overline{AD} and \overline{BC} is 5. Find the area of the shaded region.



- a) 6 b) 6.25 c) 6.5 d) 6.75 e) 7
42. If A is an acute angle and the $\sin A = \frac{2}{5}$, then $\sin 2A =$
- a) $\frac{4}{5}$ b) $\frac{1}{5}$ c) $\frac{2\sqrt{21}}{5}$ d) $\frac{2\sqrt{29}}{25}$ e) $\frac{4\sqrt{21}}{25}$
43. Suppose $x^4 + 2x^3 + kx^2 - x + 2 = (x^2 + ax + 1)(x^2 + bx + 2)$ for some real numbers a, b , and k . What is k ?
- a) -2 b) -5 c) -12 d) $1-5$ e) -23
44. Let ab, ba, cd, dc be two digit natural numbers such that $a - b = 4$ and $c + d = 8$. Find $ba - ab + cd + dc$.
- a) 28 b) 32 c) 34 d) 36 e) 52

45. Let $x < 0 < y < z$. Which one is always true?

- a) $\frac{x-y}{yz} > 0$ b) $\frac{x+y}{y+z} < 0$ c) $\frac{y+z}{xz} > 0$ d) $\frac{y-z}{x-y} > 0$ e) $\frac{z-y}{xy} > 0$

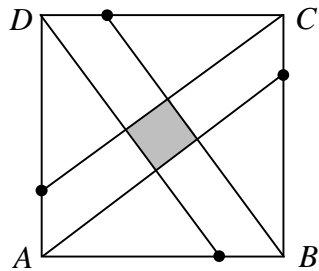
46. If you have only 5-cent stamps and 12-cent stamps, what is the largest amount of postage which you cannot exactly make?

- a) 28 b) 38 c) 43 d) 63 e) 101

47. What is the area of the quadrilateral with vertices $(0,0)$, $(3,4)$, $(-4,3)$ and $(2,11)$?

- a) 25 b) 50 c) $\frac{75}{2}$ d) 41 e) None listed.

48. The area of the square $ABCD$ is 1. Divide each side in a 2:1 ratio as shown. Find the area of the shaded region.



- a) $\frac{1}{10}$ b) $\frac{1}{11}$ c) $\frac{1}{12}$ d) $\frac{1}{13}$ e) $\frac{1}{14}$

54. Let $\frac{|x|+10}{|x|+20} = \frac{2}{3}$. Find the positive x value.

- a) 25 b) 20 c) 15 d) 10 e) 5

55. Polynomial $q(x) = x^4 + ax^3 + bx^2 - 24x + 9$ is a square of polynomial $p(x) = x^2 + cx + d$. Calculate a and b (there are two solutions). Make a sum $a + b$ for each solution. Which are these two sums?

- a) 12, 14 b) 14, 18 c) 16, 22 d) 4, 32 e) 5, -32

56. Johnny wants to get engaged to Janey but diamond rings are too expensive. So he decides to draw her a diamond instead. He graphs the set defined by $|3x| + |2y| \leq 12$. What is the area of D ?

- a) 24 b) 36 c) 48 d) 96 e) 144

57. Suppose the circle whose equation is $x^2 + y^2 + 2ax + 2by + c = 0$ has center $(-3, 1)$. If $(4, 4)$ lies on this circle, then c^{a+b} equals

- a) 3402 b) 1154 c) 4302 d) 1152 e) 2304

58. A number is divisible by 4 if and only if the last two digits of the number are divisible by 4. Which of the following statements comes closest to explaining the reason?
- a) Four is an even number and odd numbers are not divisible by even numbers.
 - b) Every number can be written as a multiple of 100 and the sum of the two digit number formed by its last two digits and 100 is divisible by 4.
 - c) Every other even number is divisible by 4; for example 24 and 28 but not 26.
 - d) It only works when the sum of the last two digits is an even number.
 - e) None listed.
59. Let x be a real number such that $2^{2^x} + 4^{2^x} = 42$. What is $\sqrt{2^{2^{2^x}}}$?
- a) 2
 - b) 4
 - c) 8
 - d) 16
 - e) 32
60. The probability is $\frac{1}{2}$ that a certain coin will turn up heads on any given toss. If the coin is to be tossed three times, what is the probability that on at least one of the tosses the coin will turn up tails?
- a) $\frac{1}{8}$
 - b) $\frac{1}{2}$
 - c) $\frac{3}{4}$
 - d) $\frac{7}{8}$
 - e) $\frac{15}{16}$
61. Of the final grades received by the students in a certain math course $\frac{1}{5}$ are A's, $\frac{1}{4}$ are B's, $\frac{1}{2}$ are C's and the remaining 10 grades are D's. What is the number of students in the class?
- a) 80
 - b) 110
 - c) 160
 - d) 200
 - e) 400.

62. Solve: $\cos \frac{\pi}{8} =$

- a) $\frac{\sqrt{2-\sqrt{2}}}{2}$ b) $\frac{\sqrt{2}}{4}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{1}{2}$ e) None listed.

63. Let $ABCD$ be a quadrilateral with sides $|AB| = 24$, $|CD| = 15$, $|AD| = 7$ and with the right angles $\angle DAB$ and $\angle BCD$. Find the area of this quadrilateral.

- a) 46 b) 2520 c) 234 d) $\sqrt{2520}$ e) $\sqrt{46}$

64. Find the value of parameter m for which the equation $2x^2 + (3 - 2m)x - m + 1 = 0$ has only one solution. Which of the following contains the value of the parameter?

- a) $\frac{\sqrt{3}}{2}, \frac{3}{2}, -\frac{3}{2}$ b) $\frac{\sqrt{3}}{4}, \frac{3}{4}, -\frac{3}{4}$ c) $\frac{1}{2}, -\frac{1}{2}, 0$ d) $\frac{1}{3}, \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}$ e) $1, \frac{1}{4}, -\frac{1}{4}$

65. Let $\frac{x}{y} - \frac{y}{x} = \frac{5x+4y}{y} + 1$. Find y with respect to x if $x + y$ does not equal 0.

- a) $-5x$ b) $-4x$ c) $-3x$ d) $3x$ e) $5x$

66. A toy car set has an orange and a green toy car. When placed on a circular track of radius 4 feet, with the orange car placed in front of the green car, the orange car goes around the track twice when it catches the green car which has gone around the track once. If the radius of the track is increased to 8 feet then how many times does the orange car go around the track before it catches the green car?

- a) 5 b) 4 c) 3 d) 2 e) Not enough information.

67. Drum X is $\frac{1}{2}$ full of oil and Drum Y , which has twice the capacity of Drum X , is $\frac{2}{3}$ full of oil. If all of the oil in Drum X is poured into Drum Y , then Drum Y will be filled to what fraction of its capacity?

- a) $\frac{3}{4}$ b) $\frac{5}{6}$ c) $\frac{11}{12}$ d) $\frac{7}{6}$ e) $\frac{11}{6}$