

1. A man buys a cycle for \$1400 and sells it at a loss of 15%. What is the selling price of the cycle?  
a) 1100            b) 1150            c) 1160            d) 1190            e) 1202
  
2. If  $f(x) = 2x - 1$  then  $f(f^{-1}(5))$  is  
a) 3                b) 5                c) -3                d) -5                e) None listed.
  
3. The sum  $1 + 2 + 3 + \dots + 50 = \underline{\hspace{2cm}}$   
a) 25(51)            b) 50(51)            c) 51(51)            d) 26(51)            e) None listed.
  
4. Suppose  $b$  is a solution to quadratic equation  $x^2 - 2x + 3 = 0$ . Evaluate  $b^3$ .  
a)  $2b + 1$             b)  $b + 3$             c)  $3 - b$             d)  $3b$             e)  $b - 6$
  
5. The hypotenuse of a right triangle is 1 inch longer than one of its legs and 8 inches longer than the other leg. What is the perimeter of the triangle?  
a) 14                b) 15                c) 17                d) 30                e) 33
  
6. Complete  $1 + (1+i) + (1+i)^2 + (1+i)^3 + (1+i)^4$  where  $i^2 = -1$ .  
a)  $3i$                 b) 1                c) 0                d)  $-4 + 5i$             e)  $6 + 2i$
  
7. If  $\log_x y = \log_y x$ , with  $x \neq y, x, y > 0$  and  $x, y \neq 1$ , what is the value of  $xy$ ?  
a) -1                b) 1                c) 2                d) -12                e) Can't determine.

8. If  $0 \leq \theta < \frac{\pi}{2}$  and  $\cos \theta = \frac{3}{5}$ , what is the value of  $\cos 3\theta$ ?
- a)  $\frac{9}{5}$                       b)  $\frac{27}{125}$                       c)  $\frac{-117}{125}$                       d)  $\frac{24}{25}$                       e) None listed.
9. What is the least number of cards which must be drawn from a deck of 52 cards to ensure that you have a 7 or a heart?
- a) 38                      b) 37                      c) 36                      d) 35                      e) 34
10. An examination consists of 3 parts: A, B, and C. In part A, a student must answer 2 of 4 questions. In part B, a student must answer 4 of 8 questions. In part C, a student must answer 3 of 5 questions. How many choices of questions does the student have?
- a) 2100                      b) 4200                      c) 2400                      d) 1500                      e) None listed.
11. Given  $f(x) = 3x + 4$ . Solve  $f(f(f(x))) + 2 = 0$ .
- a)  $-2$                       b)  $-1$                       c)  $0$                       d)  $1$                       e)  $2$
12. Find the length of the hypotenuse of a right triangle whose perimeter is 60 cm and whose area is 150 cm.
- a) 15 cm                      b) 20 cm                      c) 22 cm                      d) 25 cm                      e) None listed.
13. Solve for  $x$  if  $9^x + 9^x + 9^x = 27^{27}$ .
- a) 9                      b)  $3^{27}$                       c) 40                      d) 3                      e) None listed.
14. For what real values is  $|x| \geq x$ ?
- a)  $x > 0$                       b)  $x < 0$                       c)  $x \leq 0$                       d)  $x \geq 0$                       e) None listed.

15. A pupil's marks were wrongly entered as 83 instead of 63. Due to the average marks for the class were increased by half ( $1/2$ ), the number of pupils in the class is

- a) 10                      b) 20                      c) 40                      d) 35                      e) 73

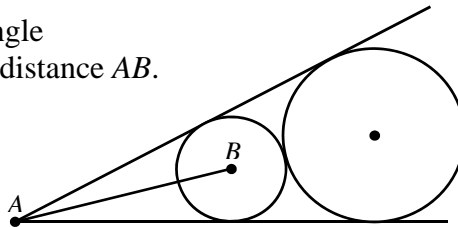
16. Let  $(a, b, 12)$  be a geometric sequence such that the sequence  $(1, a, b - 1)$  is arithmetic. Find  $a$  and  $b$ . What is their sum  $a + b$ ?

- a)  $-22$                       b)  $0$                       c)  $9$                       d)  $21$                       e)  $22$

17.  $\tan x + \cot x = 4$ . Find  $\sin(2x)$ .

- a)  $1/6$                       b)  $1/2$                       c)  $2/3$                       d)  $3/5$                       e)  $4/5$

18. The two tangent circles inscribed in an angle (see Figure) have radii 1 and 4. Find the distance  $AB$ . ( $B$  is the center of the smaller circle.)



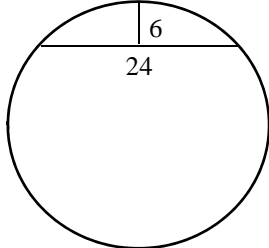
- a)  $5/3$                       b)  $5$                       c)  $17$                       d)  $17/5$                       e)  $19/5$

19. Suppose  $\frac{3}{x} + \frac{5}{y} + \frac{7}{z} = \frac{7}{3}$ . Find  $\frac{3+x}{x} + \frac{5+y}{y} + \frac{7+z}{z} =$ .

- a) 21                      b)  $3/7$                       c)  $49/9$                       d)  $5 \frac{1}{3}$                       e) No definite answer, depends on  $x, y$  and  $z$ .

20. In how many ways can the numbers 1, 2, 3, 4, 5, 6 be arranged with a sequence  $a, b, c, d, e, f$  such that  $a + f = b + e = c + d$ ?

- a) 6                      b) 24                      c) 36                      d) 48                      e) 60

21. Solve  $1^{2^3} + 1^{3^2} + 2^{1^3} + 2^{3^1} + 3^{1^2} + 3^{2^1}$ .
- a) 12                      b) 24                      c) 10                      d) 42                      e) None listed.
22. Suppose that points  $(x, y)$  with the first quadrant of the plane satisfy  $x + y \leq 7$  and  $3x + y \geq 12$ . At what point does the maximum value of  $6x + y$  occur?
- a) (4,0)                      b) (12,0)                      c) (7,0)                      d) (7,12)                      e) None listed.
23. A train 240 m long passes a pole in 24 seconds. How long will it take to pass a platform 650 m long?
- a) 65 sec                      b) 89 sec                      c) 100 sec                      d) 150 sec                      e) 200 sec
24. Solve  $\left( \frac{(469+174)^2 - (469-174)^2}{469 \times 174} \right)$ .
- a) 2                      b) 724                      c) 643                      d) 643295                      e) 4
25. Find the base to 10 fraction which is equivalent to the sum  $.4_7 + .\bar{4}_7$ .
- a)  $.8\bar{4}$                       b)  $\frac{26}{21}$                       c)  $\frac{38}{45}$                       d)  $\frac{8}{7}$                       e) None listed.
26. For a given circle we have a chord with the length 24. From the middle of this chord and perpendicular to the chord has length 6. What is the area of the circle?
- 
- a)  $225\pi$                       b) 706                      c)  $415\pi$                       d)  $385\pi$                       e) Can't determine.

27. Compute  $1 - \frac{1}{2 - \frac{1}{3 - \frac{1}{4}}}$ .

- a)  $\frac{5}{6}$       b)  $\frac{1}{4}$       c)  $\frac{7}{18}$       d)  $\frac{15}{26}$       e) None listed.

28. If  $a$  and  $b$  are positive numbers such that their average plus their geometric mean is 18, find the sum of their square roots.

- a) 3      b) 4      c) 2      d) 5      e) 6

29. On a fence are sparrows and pigeons. When five sparrows leave, there are twice as many pigeons as sparrows. Then twenty-five pigeons leave and there are now three times as many sparrows as pigeons. Find the original number of sparrows.

- a) 20      b) 30      c) 7      d) 15      e) 45

30. A number consists of two digits. If the digits interchange places and the new number is added to the original number, then the resulting number will be always divisible by

- a) 3      b) 5      c) 7      d) 9      e) 11

31. If  $\ln 3 = a$  and  $\ln 6 = b$  then  $\ln 4 =$

- a)  $2b - 2a$       b)  $a - 2b$       c)  $b - 2a$       d)  $b + 2a$       e) None listed.

32. If  $f(x) = \frac{2}{3^x + 1}$ , then the domain of  $f$  is

- a)  $\mathbb{R} - \{0\}$       b)  $\mathbb{R} - \{-1\}$       c)  $\mathbb{R}$       d)  $[0, \infty)$       e) None listed.

33. Let  $A = \{1,2,3\}$ ,  $B = \{a,b\}$ , and  $R = \{(1,a),(2,a),(2,b),(3,b)\}$ . Which one of the following facts is not true about  $R$ ?

- a)  $R$  is a relation from  $A$  to  $B$
- b)  $\text{Dom } R = \{1,2,3\}$
- c)  $R^{-1} = \{(a,1),(a,2),(b,2),(b,3)\}$
- d)  $R$  is a function from  $A$  to  $B$
- e) None listed.

34. When  $x = 4$ , which of the following is true?

- a)  $x$  is a prime and  $x$  is a square.
- b)  $x$  is a prime or  $x$  is a square
- c)  $x$  is not prime and  $x$  is not a square.
- d) All of the above.
- e) None listed.

35. Find the constants  $a$ ,  $b$ , so that the 4 lines pass through the same point.

$$x + y = -1$$

$$-x + 3y = -11$$

$$ax + by = 4$$

$$2ax - by = 2$$

- a)  $a = 1$   $b = -\frac{2}{3}$
- b)  $a = -1$   $b = \frac{2}{3}$
- c)  $a = -1$   $b = -\frac{2}{3}$
- d)  $a = 1$   $b = \frac{2}{3}$
- e) None listed.

36.  $\sin(50^\circ) \cdot \cos(20^\circ) - \cos(50^\circ) \cdot \sin(20^\circ) =$

- a)  $-\frac{1}{2}$       b)  $\frac{\sqrt{3}}{2}$       c)  $\frac{1}{2}$       d)  $-\frac{\sqrt{3}}{2}$       e) None listed.

37. Two line segments of lengths 8 and 12 intersect at  $150^\circ$  at a point where neither segment is bisected. The end points of the segments are connected by segments to form a convex quadrilateral. What is one area of the quadrilateral?

- a) 96      b) 48      c) 84      d) 24      e) Impossible to determine.

38. How many solutions to the following equation?  $(x!)^2 - 25(x!) + 24 = 0$

- a) 0      b) 1      c) 2      d) 3      e) Infinitely many.

39. Let  $a$  and  $b$  be two numbers whose average is 10. Suppose that the average of  $b$  and 10 is  $\frac{1}{2}c$ . What is the average of  $a$  and  $c$ ?

- a) 22      b) 12      c) 17      d) 6      e) 15

40. In a class, there are 15 boys and 10 girls. Three students are selected at random. The probability that one girl and two boys are selected is

- a)  $\frac{21}{46}$       b)  $\frac{25}{117}$       c)  $\frac{1}{50}$       d)  $\frac{3}{25}$       e)  $\frac{7}{30}$

41. Isosceles triangle  $ABC$  has  $AB=AC$ , base  $BC = \sqrt{10}$  and the medians from vertices  $B$  and  $C$  are perpendicular. Find the length of side  $AC$ . A median in a triangle is the line through a vertex and the midpoint of the opposite side.

- a) 5      b) 2      c)  $5/2$       d)  $2/5$       e)  $\sqrt{5}$

42. Eleven years passed between the time when my father's age and mine summed to my present age, and the time when my father's age and mine summed to its present age. How old was my father, when I was born?

- a) 20                      b) 21                      c) 22                      d) 23                      e) 24

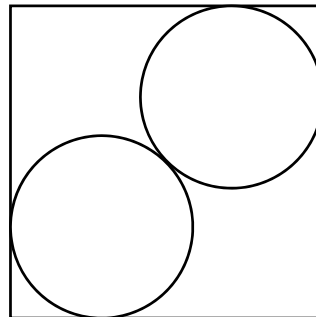
43. Suppose  $2x + y = 6$ . If  $x$  is divided by 3 and  $y$  is divided by 5 and the quotients are added, the sum is 17. What are the two numbers?

- a)  $(-98, 2021)$     b)  $(277, -548)$     c)  $(152, -298)$     d)  $(-237, 480)$     e)  $(-248, 502)$

44. Suppose  $\tan \phi = 2$  and  $\phi$  is acute. Calculate  $\sin 2\phi$ .

- a)  $7/5$                       b)  $4/5$                       c) 4                      d)  $2/3$                       e)  $3/2$

45. Two identical circles are inscribed in a  $1 \times 1$  square as shown in the figure. Find the area of both circles combined.



- a)  $\pi(2 + \sqrt{2})$     b)  $\pi(\sqrt{2/3})$     c)  $\pi(1 + \sqrt{2/2})$     d)  $\pi(3 - 2\sqrt{2})$     e)  $\pi(\sqrt{2-1})$

46. If  $a + b = 1$  and  $a^3 + b^3 = 7$ , then  $a^5 + b^5 =$

- a) 1                      b) 14                      c) 31                      d) 67                      e) 71

47. Suppose the polynomial leaves a remainder of  $5x - 7$  when divided by  $x^2 - 1$ . What is the remainder when  $P(x)$  is divided by  $x - 1$ ?

- a)  $-7$                       b) 5                      c) 1                      d)  $-2$                       e) Cannot determine.



48. Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even?

- a)  $\frac{1}{2}$       b)  $\frac{3}{4}$       c)  $\frac{3}{8}$       d)  $\frac{5}{16}$       e)  $\frac{3}{16}$

49. If  $3^{(x-y)} = 27$  and  $3^{(x+y)} = 243$ , then  $x$  is equal to

- a) 6      b) 10      c) 4      d) 0      e) 2

50. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random without replacement. What is the probability that none of the balls drawn is blue?

- a)  $\frac{10}{21}$       b)  $\frac{11}{21}$       c)  $\frac{2}{7}$       d)  $\frac{5}{7}$       e)  $\frac{5}{21}$

51. Big Jake, Cosmo D and Dinky Duke like to race each other. Big Jake can beat Cosmo D by 10 meters in a 100-meter race. Cosmo D can beat Dinky Duke by 10 meters in a 100-meter race. By how much will Big Jake beat Dinky Duke in a 100-meter race?

- a) 18 meters      b) 19 meters      c) 20 meters      d) 21 meters      e) No correct answer listed.

52. What is the length of the longest altitude in a triangle with sides of length 10, 17 and 21?

- a)  $\frac{84}{5}$       b)  $\frac{42}{5}$       c) 8      d) 21      e) No correct answer listed.

53. If  $2^x = \frac{1}{4}$  then  $3^x =$

- a)  $\frac{1}{4}$       b) 6      c)  $\frac{1}{6}$       d) 9      e)  $\frac{1}{9}$

54. The solution set of  $|x+1|+1=0$  is

- a) 1                      b)  $-1$                       c) No solution.                      d) 0                      e) All real numbers.

55. For any two functions, which one of the following facts is not necessarily true?

a)  $(f + g)(x) = f(x) + g(x)$

b)  $(f \circ g)(x) = (g \circ f)(x)$

c)  $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$ ,  $g(x) \neq 0$

d)  $f(x) + g(x) = g(x) + f(x)$

- e) All of the above.

56. In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green?

- a)  $\frac{1}{3}$                       b)  $\frac{5}{7}$                       c)  $\frac{8}{21}$                       d)  $\frac{1}{2}$                       e)  $\frac{11}{21}$

57. Suppose we are given a right triangle with a perimeter of 12 and a hypotenuse of 5. What is the area of the triangle?

- a) 12                      b) 6                      c) 16                      d) 8                      e) 7

58. Solve  $\log_2(x+2) = \log_4(9x-2)$  for  $x$ . There are two solutions. What is their sum?

- a) 3                      b) 4                      c) 5                      d) 6                      e) 7

59. Anna has read a book of 480 pages by reading the same number of pages every day. If she had read 8 pages more each day, she would have finished 3 days earlier. How many days did it take her to read the book?

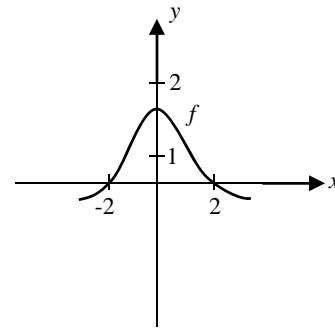
- a) 10                      b) 12                      c) 15                      d) 18                      e) 48

60. If the length of a rectangle is increased by 10% and the width is decreased by 15%, what happens to the area?

- a) Increased by 2%  
 b) Increased by 6.5%  
 c) Decreased by 2%  
 d) Decreased by 7%  
 e) Decreased by 6.5%

61. Which of the following facts is true about  $f$  ?

- a)  $f$  is increasing.  
 b)  $f(0) = 2$   
 c) It has no intercepts.  
 d)  $f(2) = 0$   
 e) None listed.



62. If  $x^3 + 3x^2 - 7x + 12 = 0$  and  $\alpha, \beta$  and  $\gamma$  are its real or complex roots, what is  $\alpha^2 + \beta^2 + \gamma^2$ ?

- a)  $-72$                       b)  $72$                       c)  $23$                       d)  $55$                       e)  $16$

63. What is the value of  $\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \dots + \frac{1}{58 \cdot 61}$ ?

- a)  $\frac{7}{3}$                       b)  $\frac{3}{5}$                       c)  $\frac{2}{3}$                       d)  $\frac{20}{61}$                       e)  $\frac{1}{3}$

64. Suppose for a certain rectangle the diagonal is 13 and the area is 60. What is the perimeter?
- a) 50                      b) 34                      c) 72                      d) 27                      e) Cannot determine.
65. Suppose that the sum of two numbers is 45 and the sum of their quotient and its reciprocal is 2.05. Find the product of the two numbers.
- a) 500                      b) 702                      c) 650                      d) 1.37                      e) Cannot determine.
66. Let  $\alpha = \overline{.10}_2$  where the subscript indicates base 2. What is the value of  $\alpha$  as a fraction in base 10?
- a)  $\frac{3}{4}$                       b)  $\frac{5}{6}$                       c)  $\frac{7}{8}$                       d)  $\frac{2}{3}$                       e)  $\frac{3}{8}$
67. Solve the equation  $\sin 4x - \cos 5x = 0$  for  $x$  in the interval  $[0^\circ, 90^\circ]$ . There are three solutions (degrees). Their sum is
- a)  $45^\circ$                       b)  $120^\circ$                       c)  $150^\circ$                       d)  $155^\circ$                       e)  $210^\circ$