Ph.D. Qualifying Examination in Statistics
Tuesday, August 31, 2004

1) Let $X_1, ..., X_n$ be independent and identically distributed (iid) from a Poisson(λ) distribution.

a) Find the limiting distribution of $\sqrt{n}\ (\ \overline{X} - \lambda\ )$.

b) Find the limiting distribution of $\sqrt{n}\ [\ (\overline{X})^3 - (\lambda)^3\ ]$.

2) Let X be a single observation from a normal distribution with mean θ and with variance θ^2, where $\theta > 0$. Find the maximum likelihood estimator of θ^2.

3) Suppose that $X_1, ..., X_n$ are iid Beta(θ, θ) random variables. (Hence $\alpha = \beta \equiv \theta$.)

a) Find a minimal sufficient statistic for θ.

b) Is the statistic found in a) complete? (prove or disprove)

4) Let $X_1, ..., X_n$ be a random sample from a distribution with pdf

$$f(x) = \frac{2x}{\theta^2},\ \ 0 < x < \theta.$$

Let $T = c\overline{X}$ be an estimator of θ where c is a constant.

a) Find the mean square error (MSE) of T as a function of c (and of θ and n).

b) Find the value c that minimizes the MSE. Prove that your value is the minimizer.

5) Suppose that $X_1, ..., X_n$ are iid Bernoulli(p) where $n \geq 2$ and $0 < p < 1$ is the unknown parameter.

a) Derive the UMVUE of $\nu(p)$, where $\nu(p) = e^2(p(1-p))$.

b) Find the Cramér Rao lower bound for estimating $\nu(p) = e^2(p(1-p))$.

6) Suppose X is an observable random variable with its pdf given by $f(x)$, $x \in R$. Consider two functions defined as follows:

$$f_0(x) = \begin{cases} \frac{3}{64}x^2 & 0 \le x \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

$$f_1(x) = \begin{cases} \frac{3}{16}\sqrt{x} & 0 \le x \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

Determine the most powerful level α test for $H_0 : f(x) = f_0(x)$ versus $H_a : f(x) = f_1(x)$ in the simplest implementable form. Also, find the power of the test when $\alpha = 0.01$.