Ph.D. Qualifying Exam in Topology

Fall 2000

Instructions: Do as many of the ten problems below as you can. Please use a separate sheet of paper for each problem. If a problem takes more than one page number the pages. Do not use the back of a page. Good luck!

1. (Answer 5 of the 9, for 2 points each.)
   a) Define cardnality.
   b) State the Well-orderong Theroem.
   c) State the Axiom of Choice.
   d) State the Maximum Principle.
   e) Define metric space.
   f) Define quotient map.
   g) Define m-dimensional manifold.
   h) Define strong deformation retract.
   i) Define covering map.
2. Prove that every metrizable space is normal.
3. Prove that $\mathbb{Q}$, the rational numbers, is not locally compact.
4. State and prove the Lebesgue Number Lemma.
5. Give an example showing that the product of two quotient maps need not be a quotient map.
6. Show that every uncountable subset of the real line has a limit point.
7. Prove that the connected subsets of the real line are intervals (or one-point sets).
8. Prove that every one-point subset of a Hausdorff space is closed.
9. Let X and Y be topological spaces; let $x \in X$ and $y \in Y$. Prove that

$$\pi_1(X \times Y, (x, y)) \cong \pi_1(X, x) \times \pi_1(Y, y).$$

10. Prove that if $p : E \longrightarrow B$ is a covering map, then p is an open map.