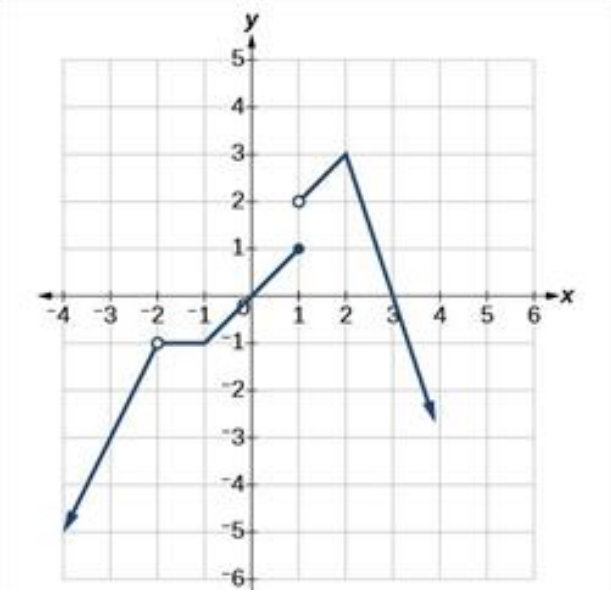
Final Exam Math 150 Fall 2023 You must show ALL work for credit.

1. (20 points) Use the graph of f below to state the value of each quantity if it exists:



a) $\lim_{x\to 1-} f(x) =$ _____

b) $\lim_{x\to 1+} f(x) =$ _____

c) $\lim_{x\to 1} f(x) =$ _____

d) $\lim_{x\to -2} f(x) =$ _____

e) $f(-2) =$ _____

f) $\lim_{x\to 2} f(x) =$ _____

g) $\lim_{x\to 3} f(x) =$ _____

h) List the x- value(s) where the function is NOT differentiable: $x =$ _____

2. (15 points) Find each limit. If the limit does not exist, say so and state why.

a) $\lim_{x\to -5} \frac{25-x^2}{x+5}$

b) $\lim_{x\to -\infty} \frac{\sqrt{16x^2+12x+49}}{7+4x}$

c) $\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1}$

3. (6 points) Find the derivative of the function using the **<u>definition of the derivative</u>** for
f(x) = $\frac{3}{2x+1}$

4. (25 points) Find each derivative. **<u>DO NOT SIMPLIFY</u>** your answers.

a) f(x) = $e^{-2x} + 7 \ln x + e^3$

b) g(x) = $3 \cos^2(2x) + 7\sec(2x)$

c) h(x) = ($\sqrt{3x+2} + 5x^2$) ($\tan(x) + 5x^3$)

d) p(x) = $\frac{x^3-x}{2x+\cos(x)}$

e) F(x) = $\int_3^{4x^2} \sqrt{2t^3+2}\ dt$

5. (8 points) Compute the derivative using **logarithmic differentiation**. DO NOT SIMPLIFY!!!

y = $\frac{(\sin(x))^x}{(2x^2+5)^9}$

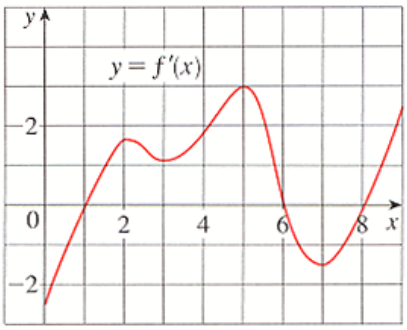
6. (8 points) Use **implicit differentiation** to find $\frac{dy}{dx}$ if $\sin(2x^2y) + 3x^4 + 5y^2 = 81$

7. (10 points) Find the equation of the line tangent to $f(x) = (3x - 4)^3$ where x =2. Write your final answer in **slope-intercept form**.

8. (7 points) Find the **critical numbers** of $f(x) = \sqrt{3}\ x + 2 \sin x$ on the interval $[\ 0, 2\pi\ ]$

9. (8 points) Find the **absolute extrema** of $f(x) = 2x + \frac{8}{x}\ \ on\ \ [1, 5]$

10. (12 points) Suppose that the **DERIVATIVE** f' of a function has the graph. (Assume the function f is defined only for $0 < x < \infty$.) Find the following:



a) Open interval where f is increasing:_____

b) State the x- coordinate of all extrema and label as max/min.

Max at $x =$ _____

Min at $x =$ _____

c) Open interval(s) where f is concave up:_____

d) State the x- coordinate of all Inflection points. $x =$ _____

11. (12 points) A $20\ ft.$ ladder is leaning against a vertical wall on level ground. The bottom of the ladder is pulled from the wall at a rate of $\frac{5}{2}$ feet per second.

a) When the top of the ladder is $16\ ft.$ above the ground how fast is it falling at this time? Give an exact answer.

b) How fast is the area of the triangle formed by the ladder, the wall and the ground changing at this time? Be sure to indicate whether the area is increasing or decreasing. Give an exact answer.

12. (15 points) Let G(x) = $(x-3)^3 - 27x$, use calculus to find the following:

a) Find the intervals G(x) is increasing.

b) Find the relative extrema of G(x) as ordered pairs and indicate whether they are maxima or minima.

c) Find the interval(s) G(x) is concave up.

d) Find any inflection points as ordered pairs.

13. (30 points) Integrate the following.

a) $\int (2x^2 + 4)^2 \, dx$

b) $\int (\sin(3x) + sec^2(4x) + e^{7x}) \, dx$

c) $\int \frac{e^{3x}+1}{e^{3x}+3x} \, dx$

d) $\int \frac{2x^3+7x+13}{x^2} \, dx$

e) $\int 6xe^{2x^2}dx$

14. (14 points) Evaluate the following definite integrals

a) $\int_2^6 2x\sqrt{x-2}\,dx$

b) $\int_0^{\pi/6} 4\,sin(2x)\;\;dx$

15. (10 points) Find the dimensions of the rectangle of largest area which can be inscribed in the ellipse with equation $4x^2 + y^2 = 16$