CONTROL THEORY ABSTRACTS

Mohamed Ali Belabbas
University of Illinois, Urbana-Champaign

Title: The geometry of optimal sensor and actuator design.

Abstract:
Consider the estimation of the state of such processes from linear, noisy measurements; the Kalman filter is known to be the minimum mean square error estimator when the measurement noise is Gaussian. We address here how to design the sensor that minimizes the error afforded by the Kalman filter. This problem of optimal sensor design, which is almost as old as the Kalman filter itself, is however not convex. As a consequence, many ad hoc methods have been used over the years to solve it. We show in this talk how a geometric approach allows us to characterize and obtain the optimal design exactly. This optimal design yields the lowest possible estimation error from linear measurements with a fixed signal to noise ratio. The dual problem of optimal actuator design for linear quadratic control is also addressed.

Arash Komaee
Southern Illinois University Carbondale
akomaee@siu.edu

Title: Potential Canals for Control of Nonlinear Stochastic Systems in the Absence of State Measurements

This talk considers design of an open-loop control to direct the state of a nonlinear stochastic system from a random initial state toward a targeted stable equilibrium generated by a predefined constant control. Under this constant control, the system is assumed to have multiple stable equilibria, so that starting from a random initial state, the system can settle eventually at any of these equilibria, not necessarily the targeted one. To direct the system toward the targeted equilibrium, an initial phase of dynamic (time-varying) control is needed before application of the constant control. The concept of potential canal is introduced in this paper to develop a methodology for design of such dynamic control. This methodology is primarily intended to be used in directed self-assembly, a technology for ordering charged nanoparticles into desired nanostructures by manipulating external electrical fields. The features of the developed control are demonstrated for directed self-assembly in one dimension.

Nicholas LaRacuente, James O’Dwyer
University of Illinois at Urbana-Champaign

Title:
Hidden Variable Theories for Complex Systems without Prior Knowledge of Mathematical Form

Many complex systems, including those commonly found in ecology and systems biology, inherently involve more variables than would be practical to measure directly. Hidden variable theories of such phenomena would be mechanistically correct and optimally predictive, but we often lack a priori knowledge of the form and extent of these unseen interactions. We address these challenges with a new method inspired by recent developments in equation-free forecasting techniques including Empirical Dynamic Modeling and Sparse Identification of Non-linear Dynamics. We show how it is often possible, knowing very little about a system that generated a time series, to make quantitative predictions of future dynamics and obtain information that can help us construct a more traditional mechanistic model. We consider real-world time series from traditional predator-prey systems, more complicated environments and even socioeconomic complex systems. We also begin to address limitations of previous equation-free techniques, including generalization to longer timescales and non-stationary processes.

Jun Liu

Jackson State University

Talk title:
Efficient Time Domain Decomposition Algorithms for Time-Dependent PDE-Constrained Optimization Problems

Abstract:
Optimization with time-dependent partial differential equations (PDEs) as constraints appear in many engineering applications. The associated first-order necessary optimality system consists of one forward and one backward time-dependent PDE coupled with optimality conditions. An optimization process by using the one-shot method determines the optimal control, state and adjoint state at once, while, with the cost of solving a large scale, fully discrete optimality system. Hence, such one-shot method could easily become prohibitive when the time span is long or a small time step is taken. To overcome this difficulty, we propose in this paper several time domain decomposition algorithms for improving its computational efficiency. In these algorithms, the optimality system is split into many small subsystems over a much smaller time interval, which are coupled by appropriate continuity matching conditions. Both one-level and two-level multiplicative and additive Schwarz algorithms are developed for iteratively solving the decomposed subsystems in parallel. In particular, the convergence of the one-level multiplicative and additive Schwarz algorithms without overlap are proved. The effectiveness of our proposed algorithms is demonstrated by both 1D and 2D numerical experiments, where the developed two-level algorithms show very scalable convergence rates with respect to the number of subdomains.

Phasic response motifs are optimal for persistent detections

Anirban Nandi, ShiNung Ching, Washington University in St. Louis

Recent evidence suggests that the stimulus-evoked responses of early olfactory neurons dichotomize into phasic and tonic temporal motifs. Presumably, such responses mediate the eventual decoding of the stimulus identity. However, the precise mechanisms of such decoding and, in particular, the advantages of having these different sets of temporal dynamics is not well understood. Here, we perform a computational study to disentangle these different sensory-evoked responses. We consider a Two Alternative Detection (TAD) paradigm in which sensory 'evidence' towards each of two stimulus compositions is integrated within a drift-diffusion model (DDM) framework and a detection is made in favor of one of the compositions. Representing the neural response to sensory inputs as the evidence driving the latent state of the DDM, we investigate the optimality of these neural motifs with regards to different drift dynamics.

We examined stimulus evoked responses of projection neurons in the locust antennal lobe circuit that receive direct sensory input from the olfactory receptor neurons. We observed that evoked responses contain two major, mutually exclusive motifs: spiking activity during stimulus presentation followed by inhibition ('ON' type) or inhibition during the stimulus presentation followed by pronounced phasic activation after stimulus termination ('OFF' type). The ON, OFF clusters can be further sub-classified according to the magnitude of their phasic parts, as shown in Figure 1a.

We then asked whether such motifs might be optimal conveyors of sensory 'evidence' within a DDM decoding scheme in a proposed ***Persistent response paradigm***, where the objective of the neural response is to retain the sensory percept until the stimulus is withdrawn. For this, we considered a quadratic cost function of the form

$$\begin{aligned} &\underset{\mathbf{y}}{\text{minimize}} \quad \mathbb{J}(\mathbf{y}) = \int_0^{\tau} \frac{1}{2}[(\boldsymbol{\nu}(t)-\mathbf{z})^T\mathbf{Q}(t)(\boldsymbol{\nu}(t)-\mathbf{z}) + \mathbf{x}(t)^T\mathbf{R}_1(t)\mathbf{x}(t) + \mathbf{y}(t)^T\mathbf{R}_2(t)\mathbf{y}(t)]\mathrm{d}t \\ &\text{subject to} \quad \dot{\boldsymbol{\nu}}(t) = f(\boldsymbol{\nu},\mathbf{x}), \dot{\mathbf{x}} = \mathbf{y},\ \boldsymbol{\nu}(0) = \boldsymbol{\nu}_0,\ \mathbf{x}(0) = \mathbf{x}_0,\ \mathbf{Q}(t), \mathbf{R}_1 \geq 0, \mathbf{R}_2 > 0. \end{aligned} \tag{1}$$

where $\boldsymbol{\nu} \in \mathbb{R}^2$ is the latent state, $\mathbf{x} \in \mathbb{R}^M$ is the neural response from M neurons. Here, $f(\boldsymbol{\nu},\mathbf{x})$ is the drift function and $\mathbf{z}$ represents the target representation, within which is embedded the detection threshold as well as the suppression of competing processes. The cost over a horizon τ penalizes: (i) the latent state's departure from the desired threshold (thus yielding a persistent detection); (ii) the energy and velocity $\mathbf{y}$ of the neural response (via $\mathbf{Q}(t)$, $\mathbf{R}_1(t) \geq 0$, $\mathbf{R}_2(t) > 0$ respectively).

In Figure 1c we plot the optimal motif for the equilibrium model ($f(\boldsymbol{\nu},\mathbf{x}) = \mathbf{A}\boldsymbol{\nu} + \mathbf{b}\mathbf{x}$, where $\mathbf{A} \in \mathbb{R}^{2\times 2}$, $\mathbf{b} \in \mathbb{R}^{2\times M}$ and $\sigma(\mathbf{A}) \in \mathbb{R}_{<0}$) without noise. The first motif (red) displays a phasic response following stimulus onset, followed by a tonic component during stimulus persistence. The second motif (blue) displays a relatively tonic inhibition during stimulus persistence, and a phasic response on stimulus withdrawal. These motifs are consistent with the ON and OFF clusters observed in our data (Figure 1a). It should be noted that additive Gaussian noise would not change the optimality of these motifs. Our results indicate how neural responses in early sensory networks may enable time-optimal formation and maintenance of representations of a persistent stimulus.

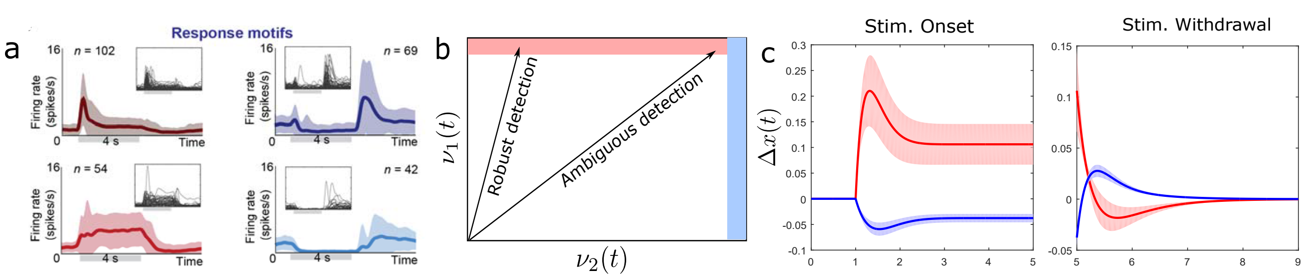

Figure 1: *(a) Neural responses in olfactory projection neurons dichotomize into two response classes: Phase/Tonic ON (left) and OFF (right). (b) We consider a DDM framework with an objective function designed to allow thresholds to be hit and held with minimal ambiguity due to competing latent variables. (c) Optimizing persistent detection according to the cost* (1) *in an equilibrium DDM results in production of similar response motifs.*

Mohammad Sayeh
Southern Illinois University Carbondale

Title: Dynamics of Photorefractive Grating

Abstract: The photorefractive grating generated by two coupled waves produces a rich spatiotemporal dynamics. In this talk, a set of solutions of the dynamic grating equations are presented and analytically solved in the form of the Jacobi's Elliptic functions. This dynamics interestingly relates to that of spinning top.

Fractional Calculus Abstracts

Udita N. Katugampola

University of Delaware, Newark DE 19716.

Title: New fractional integral with applications to time-fractional Porous Medium Equation (PME).

Abstract: In this talk we study some properties of a generalized fractional integral which unifies six familiar fractional integrals in to one form and is given by

$$\left({}^{\rho}\mathcal{I}^{\alpha,\beta}_{a+;\eta,\kappa}f\right)(x) = \frac{\rho^{1-\beta}x^{\kappa}}{\Gamma(\alpha)}\int_a^x \frac{\tau^{\rho\eta+\rho-1}}{(x^{\rho}-\tau^{\rho})^{1-\alpha}}f(\tau)\mathrm{d}\tau,$$

where $0 \leq a < x < b \leq \infty$. We obtain a series representation of this integral along with asymptotic expansion and use it to solve a *time-fractional porous medium equation* in a subdiffusive case $(0 < \alpha < 1)$ of the form

$$\begin{cases} \partial_t^{\alpha} u(x,t) = \left(D_0 u^m(x,t) u_x(x,t)\right)_x \\ u(0,t) = C, \quad u(x,0) = 0, \end{cases}$$

where ∂_t^{α} is the *Riemann-Liouville* fractional derivative of order α. We show that *Erdélyi-Kober* type integral naturally appears in the model equation which then be replaced by the integral in question. We study several cases of m to illustrate the applicability of this approach. Also, we derive two new identities of Taylor-type which can be used in broader sense.

Ryan M. Evans[1], Udita N. Katugampola[2], David A. Edwards[2]

[1]National Institute of Standards and Technology, Gaithersburg, MD 20899.

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Title: Applications of fractional calculus in solving Abel-type integral equations: Surface-volume reaction problem

Abstract: In this talk we consider a class of time-fractional partial differential equations of fractional order, motivated by an equation which arises as a result of modeling *surface-volume reactions* in optical biosensors. Surface-volume reactions occur in blood-clotting, DNA-repairing and drug absorption. We notice that one of the model equations takes the following form

$$C(x,0,t) = 1 - \frac{D_a}{3^{\frac{1}{3}}\Gamma(2/3)} \int_0^x \frac{\partial B}{\partial t}(\nu,t) \frac{d\nu}{(x-\nu)^{-2/3}}$$

which is a fractional integral of *Riemann-Liouville* type. Here B and C are concentrations of bound ligand and unbound ligands, respectively. We solve these equations by employing techniques from fractional calculus; several examples are discussed. Furthermore, for the first time, we encounter an order of the fractional derivative other than $\frac{1}{2}$ in an applied problem further exposing the true nature of fractional calculus.

Om P. Agrawal
Professor and Distinguished Scholar
Mechanical Engineering and Energy Processes
Southern Illinois University
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Abstract

The concept of fractional derivatives was introduced almost at the same time that the concept of integer order derivatives was introduced. In recent years, the concept and applications of fractional derivatives have received considerable interest in almost every area of science, engineering, applied mathematics, bioengineering and economics. It has been demonstrated that fractional derivatives based models provide more accurate models of many systems than their integer order models do. In this talk, several definitions of fractional derivatives used in the literature will be introduced. Integer order derivatives are defined using local information. In contrast, global information, namely past history of the system is needed to define a fractional derivative. As a result, fractional derivatives turn out to be useful in modeling systems with memory.

In this talk, several applications will be discussed where fractional derivatives arise. These applications include tautochrone problem, modeling of a disk brake, viscoelastic system, lossy transmission line and lossy capacitor, and vibration of a plate submerged in a fluid. Analytical results will be compared with experimental results where possible. Many researchers avoid models containing fractional derivatives because they assume that these models require a strong theoretical background in fractional calculus. In this talk, it will be demonstrated that tools can be developed to hide mathematical details necessary to solve equations involving fractional derivatives. Applications of these tools in solving fractional differential equations and fractional control problems will also be demonstrated. Finally, some of the areas where fractional derivatives are being applied today will be mentioned. Although considerable research has been done on applications of fractional derivatives, much remains to be done. It is hoped that this presentation will encourage the listener to start doing research in the area of fractional derivatives.

Charles Delman - Eastern Illinois University

Title: Foliations, Heegaard-Floer Homology, and Dehn Surgery on Knots

Joint work with Rachel Roberts

Abstract:

Heegaard-Floer Homology, introduced by Peter Ozsváth and Zoltán Szabó, is a powerful invariant that associates, to any rational homology sphere, a vector space over the finite field of two elements. An L-space is a homology 3-sphere whose Heegaard-Floer homology has minimal rank; lens spaces are examples (hence the name). Results of Ozsváth - Szabó, Eliashberg - Thurston, and Kazez - Roberts show that a manifold admitting a taut, co-orientable foliation cannot be an L-space. Let us call such a manifold *foliar*. Ozsváth and Szabó have asked whether or not the converse is true for irreducible 3-manifolds.

Restricting attention to manifolds obtained by Dehn surgery on knots in S^3, we posit the following:

(Classical) L-space Surgery Conjecture. *Suppose $\kappa \subset S^3$ is a knot in the 3-sphere. Then a manifold obtained by Dehn filling along κ is foliar if and only if it is irreducible and not an L-space.*

Using generalized surface decomposition techniques that build on earlier work of Gabai, Menasco, Oertel, and the authors, we prove that both alternating knots and Montesinos Knots satisfy the L-space Surgery Conjecture. We believe these techniques will prove fruitful in the further study of taut foliations in 3-manifolds.

Gregory Galperin
Eastern Illinois University

TITLE: *The John Conway sequence, the associated 92-dimensional operator, and the Conway's Cosmological Constant*

ABSTRACT: One of the most intriguing sequences, the so-called "*Look and Say Sequence*" or the *C-sequence* is Conway's: 1, 11, 21, 1211, 111221, 312211, 13112221, ... It is created by the *audio-active operator* $\mathcal{A}$ which transforms each term to the next one by "reading" the digits of the previous term as follows:

$$1 \Rightarrow one\ 1 \Rightarrow two\ 1's \Rightarrow one\ 2, one\ 1 \Rightarrow one\ 1, one\ 2, two\ 1's \Rightarrow \ldots .$$

The Conway's fundamental theorem below answers, in particular, to the following questions:

Can a C-number has 2017 digits? Will 4 *show up as a digit in some C-number?*
Is there a C-number that contains ...333...? Is the whole C-sequence periodic or not?
How fast does the C-numbers grow? What are the densities of the C-digits?

Cosmological Theorem. Each C-number contains only digits 1, 2, and 3. If d_n stands for the length of the nth C-number, then:

(a) The limit $\lambda := \lim_{n\to\infty} d_{n+1}/d_n$ exists;
(b) The number λ is the unique positive real root of a special polynomial of degree 71;
(c) The densities of the C-digits converge to $\nu(1) \approx 0.46$, $\nu(2) \approx 0.35$, and $\nu(3) \approx 0.19$.

The number $\lambda \approx 1.303577269...$ is known as *Conway's Cosmological Constant*.

The crucial point in Conway's proof is that each C-number is made up of one or more of 92 "basic" non-interacting strings, or "*audioactive molecules*", called him as the basic 92 chemical elements, from Hydrogen H to Uranium U.

The audio-active operator $\mathcal{A}$ generates a 92-dimensional operator $\mathcal{T}: \mathbb{R}^{92} \to \mathbb{R}^{92}$ of frequencies of these molecules. Operator $\mathcal{T}$ has a 71-dimensional subspace on which it is irreducible, and Conway's constant λ is actually the maximal positive eigenvalue of the operator $\mathcal{T}$.

Louis H Kauffman

University of Illinois at Chicago

Majorana Fermions, Braiding and Quantum Computing

Abstract

We will discuss the mathematics of Majorana Fermions and the structure of representations of the Artin Braid group that are associated with them. We will discuss how one class of representations is related to the Temperley Lieb algebra, and another class of representations is related to a Hamiltonian constructed from the Bell-Basis solution of the Yang-Baxter equation, and the relationship of this Hamiltonian with the Kitaev spin chain.

H. Makaruk, J. Langenbrunner, R. Owczarek

Los Alamos National Laboratory

"A Dynamical System of Deuterium –Tritium Reaction", LA- UR-17-23290

Abstract

Reactive DD or DT (deuterium plus tritium) plasma behaves like a dynamical system. Flux versus time derivative of the flux trajectories wander around an apparent fixed point. We construct a Hamiltonian of this system. Hamiltonian formalism allows us to obtain:

- potential for the system, and its various plots.
- constant of the motion analogous to the energy.
- trajectories in the phase space, and their plots.
- general analytical solution in an implicit form.
- explicit solutions in a quadratic approximation

Robert Owczarek
University of New Mexico

Title: Applications of Chebyshev polynomials in knot theory

Abstract
I will introduce in a new way the Chebyshev polynomials of two variables and then discuss intriguing relationships between these polynomials, Chebyshev polynomials of the first and the second kind, and Lucas and Fibonacci polynomials on one side, and their further relationships with the Jones polynomial for knots defined using the Kauffman bracket, and its generalization inspired by the relationhips.

Indu Satija,

George Mason University

A Tale of Three Fractals

The Hofstadter Butterfly, the Integral Apollonian Gaskets and the Pythagorean Triplets

The Hofstadter buttery is a fascinating two-dimensional spectral landscape – a graph of allowed energies of an electron moving in a two- dimensional crystal in a traverse magnetic field. It is a quantum fractal madeup of integers of topological origin, describing topological states of matter known as the integer quantum Hall states. In my talk, I will discuss how this quantum fractal is related to the integral Apollonian gaskets and the Pythagorean triplets. These mappings reveal numerous number theoretical patterns describing asymptotic scalings of the butterfly, all encoded in the eigenvalues of the product of Hall matrices. A fascinating result is an emergence of a three fold symmetry of mysterious nature, embedded in the kaleidoscopic images of the D3 symmetric Apollonian. Some of these ideas are described in my recent book, ”Butterfly in the Quantum World” with contributions by Douglas Hofstadter (IOP Concise, Morgan and Claypool publication, 2016).

Mohammad Sayeh

Southern Illinois University Carbondale

Title: Fractional Fourier Transform and Optics

Abstract: Fractional Fourier Transform (FrFT) is a generalized Fourier transform with a chirp function as its kernel. FrFT can be implemented using optical fiber or lens . Applications of this transform will be discussed which include chirp filtering and virtual

Prof. Matthias Weber
Indiana University

Björling Surfaces with Prescribed Geometry

We present a new method to construct explicit, regular minimal surfaces in Euclidean space that are defined on the entire complex plane with controlled geometry. More precisely we show that for a large class of planar curves (x(t),y(t)) one can find a third coordinate z(t) and normal fields n(t) along the space curve c(t) = (x(t), y(t), z(t)) so that the Björling formula applied to c(t) and n(t) can be explicitly evaluated. We give many examples.

MATHEMATICAL BIOLOGY ABSTRACTS

Craig Collins
Murray State University

TITLE: Optimal Control and Tumor Modeling

ABSTRACT:

The techniques of mathematical modeling are valuable tools for the medical community, particularly in the field of cancer research. Using models that simulate the interactions between tumor cells, chemotherapy drugs, and various components of the immune system, researchers can gain useful information about the dynamics of these complex biological systems. The application of optimal control theory to such models provides insight into the potential benefits of particular treatment regimens. In this talk, we will examine a particular strategy of combining chemotherapy with immunotherapy, as well as other scenarios.

Dr. Christina J. Edholm
Post-Doc Teaching Associate
Department of Mathematics
University of Tennessee

In light of the Ebola outbreak in 2014, we worked on an Ebola model during our South Mrica Mathematical Sciences Association Masmau program in 2014 and 2015. Our model partitions the population into those who take precautions against contracting the disease and those who do not. We consider new infections arising in both hospital settings as well as in the community, and include transmission from dead bodies and the environment. Our goal is to illustrate role of education in limiting a potential future Ebola outbreaks in Sudan using data and modeling. We considered implications of a new strain with respect to different death rates and recovery rates.

Rong Fan - Southern Illinois University Carbondale

Random Walk with Barriers and Telegraph Equation with Boundaries

Abstract

Reaction-diffusion equations are utilized to model organisms' dispersal and their population dynamics. However, some empirical studies suggested that details of organism dispersal process should be considered to explain some phenomena that cannot be explained by simple reaction-diffusion models. So, we studied telegraph equation and biased telegraph equation which contain details of organism dispersal process. Usually, the analytic solution to these two telegraph equations are hard to be obtained and parameters in the equations are also hard to be estimated. Random walks can be used as a tool to overcome these difficulties. We solved the two equations with absorbing boundaries and then simulated the solutions by various random walks with barriers. In this presentation, some numerical simulations will be provided to illustrate that the distributions of simulated random walks are consistent with the solutions of these two equations. After that, I'll talk about how to estimate the parameters of the telegraph equations and a little bit of our future work.

Nicholas LaRacuente, James O'Dwyer
University of Illinois at Urbana-Champaign

Title: Hidden Variable Theories for Complex Systems without Prior Knowledge of Mathematical Form

Many complex systems, including those commonly found in ecology and systems biology, inherently involve more variables than would be practical to measure directly. Hidden variable theories of such phenomena would be mechanistically correct and optimally predictive, but we often lack a priori knowledge of the form and extent of these unseen interactions. We address these challenges with a new method inspired by recent developments in equation-free forecasting techniques including Empirical Dynamic Modeling and Sparse Identification of Non-linear Dynamics. We show how it is often possible, knowing very little about a system that generated a time series, to make quantitative predictions of future dynamics and obtain information that can help us construct a more traditional mechanistic model. We consider real-world time series from traditional predator-prey systems, more complicated environments and even socioeconomic complex systems. We also begin to address limitations of previous equation-free techniques, including generalization to longer timescales and non-stationary processes.

Suzanne Lenhart
University Tennessee Knoxville

Title Modeling of Johne's Disease in Dairy Cattle

Abstract: Johne's disease in dairy cattle is a chronic infectious disease in the intestines caused by the baciili, *Mycobacterium avium ssp. paratuberculosis*. We have modeled this disease with several approaches to illustrate different features. A system of difference equations represented an epidemiological situation in dairy farm to compare the effects of two types of diagnostic tests. Then an agent-based model at the farm level was developed to see the effects of stochasticity, Lastly, a PDE/ODE model illustrated a novel way to link a within-host model with an epidemiological model.

Bingtuan Li
University of Louisville

Title: Persistence and Spreading Speeds of Spatial Population Models with A Shifting Habitat

Abstract: We discuss mathematical models in the forms of reaction-diffusion equations and integro-difference equations that describe the spatial dynamics of a species in a shifting habitat along which the species growth increases. We give conditions under which the species disperses to a region of poor quality where the species eventually becomes extinct. We show that when the species persists in the habitat, the rightward spreading speed and leftward spreading speeds are determined by c, the speed at which the habitat shifts, as well as the dispersal and growth of the species. We demonstrate that it is possible for a solution to form a two-layer wave, with the propagation speeds of the two layers analytically determined.

Modeling the population dynamics and community impacts of *Ambystoma tigrinum*

Maeve L. McCarthy
Murray State University

Phenotypic plasticity is the ability of an organism to change its phenotype in response to changes in the environment. General mathematical descriptions of the phenomenon rely on an abstract measure of "viability" that, in this study, is instantiated in the case of the Tiger Salamander, Ambystoma tigrinum . This organism has a point in its development when, upon maturing, it may take two very different forms. One is a terrestrial salamander (metamorph) that visits ponds to reproduce and eat, while the other is an aquatic form (paedomorph) that remains in the pond to breed and which consumes a variety of prey including its own offspring.

A seven dimensional nonlinear system of ordinary differential equations is developed. One parameter in the model controls the proportion of juveniles maturing into terrestrial versus aquatic adults . Solutions are shown to remain non-negative. A sensitivity analysis and equilibrium analysis of model parameters demonstrate that morphological choice is critical to the overall composition of the Ambystoma population. Optimal responses change with spatiotemporal variation, which is consistent with other phenotypically plastic systems. Two competing hypotheses for the conditions under which metamorphosis occurs are examined in light of the model and data from an Ambystoma tigrinum population at Mexican Cut, Colorado. The model clearly supports one of these over the other for this data set.

Multistationarity in reaction networks models

MAYA MINCHEVA
NORTHERN ILLINOIS UNIVERSITY Department of Mathematical Sciences Dekalb, IL, USA
mmincheva@niu.edu

Mathematical models in biology are often available in the form of or-dinary differential equations. Multistationarity or the existence of sev-eral steady states is an ubiquitous phenomenon in biochemical reaction models. A computational procedure for the existence of several positive steady states will be presented. The input is a system of ordinary differ-ential equations whose behavior depends on a set of parameters and the output is a single polynomial. The sign of this polynomial determines if multiple steady states exist and for what parameter values. Often we can obtain parameter inequalities that ensure the sign of the polynomial resulting in multiple (unique) steady states. These parameter inequalities in turn define parameter regions where multiple (unique) steady states exist.

This is a joint work with Carsten Conradi, Elisenda Feliu and Carsten Wiuf.

A simple spatiotemporal epidemic model for the Zika virus dynamics: computational simulations

Tiago Y. Miyaoka, João F. C. A. Meyer, Juliana M. R. Souza
University of Campinas, Brazil

March 2017

Abstract

Zika virus is a serious worldwide concern nowadays, mainly due to microcephaly in babies born from mothers that have been infected during pregnancy. The disease vector, the aedes aegypt mosquito, is also responsible for the spread of other diseases, such as Chikungunya and Dengue, this last with a considerable mortality rate. In this work, a simple compartmental SIS model with spatial diffusion is considered to study the dynamics of the Zika virus in human population, without considering explicitly the mosquito population. Let S and I represent the susceptible and infected human populations. Then we have the following Partial Differential Equations (PDE) model, for $(x, y, t) \in \Omega \times [0, T]$:

$$\begin{cases} \dfrac{\partial S}{\partial t} - \nabla \cdot (\alpha_S \nabla S) = -\beta SI + r\,(S+I)\left(1 - \dfrac{(S+I)}{\kappa}\right) + \sigma I. \\ \dfrac{\partial I}{\partial t} - \nabla \cdot (\alpha_I \nabla I) = \beta SI - \sigma I - \mu I. \end{cases} \tag{1}$$

with proper initial and boundary conditions. Here, α_S and α_I denotes diffusion coefficients for the populations, β the disease infection rate, which considers implicitly the mosquito vector action, r the growth rate, κ the support capacity, σ the recover rate, and μ the infected mortality rate.

Restricting this model to the temporal variable, one obtains an Ordinary Differential Equations (ODE) model, which stability analysis can give us information about equilibria points related to extinction and persistence of the disease. We analyzed these equilibria points with a Monte Carlo generation of the model's parameters, obtaining the most probable endemic configuration. With this configuration, we simulated numerical solutions of the PDE model (1), using Finite Elements and Crank-Nicolson methods.

Using a change of scale, we simulated a situation in a spatial region equivalent to the state of São Paulo, Brazil. With constant parameters, the numerical solutions of the PDE model converge to solutions of the ODE model. Moreover, the simulations show that the simple model studied is capable of present results that may be useful in a better understanding of the Zika virus spread.

David Murrugarra
University of Kentucky

Title: Control Methods for Discrete Networks

Abstract: Many problems in biomedicine and other areas of the life sciences can be characterized as control problems, with the goal of finding strategies to change a disease or otherwise undesirable state of a biological system into another, more desirable, state through an intervention, such as a drug or other therapeutic treatment. The identification of such strategies is typically based on a mathematical model of the process to be altered through targeted control inputs. This talk focuses on processes at the molecular level that determine the state of an individual cell, involving signaling or gene regulation. The mathematical model type considered is that of Boolean networks. The potential control targets can be represented by a set of nodes and edges that can be manipulated to produce a desired effect on the system.
This talk presents a method for the identification of potential intervention targets in Boolean molecular network models using algebraic techniques. The approach exploits an algebraic representation of Boolean networks to encode the control candidates in the network wiring diagram as the solutions of a system of polynomials equations, and then uses computational algebra techniques to find such controllers. Additionally, a formula, based on the properties of Boolean canalization, for estimating the number of changed transitions in the state space of the system as a result of an edge deletion in the wiring diagram will be discussed.

Zoi Rapti
University of Illinois Urbana-Champaign

Title: Epidemic and dispersal models based on PDEs: what can be said about their linear spectrum

Abstract: We will talk about the spectral properties of certain classes of PDEs with applications to epidemic and animal dispersal models. We will present results that show how PDEs can be effectively approximated by ODEs with the same behavior in terms of R0 (the basic reproductive rate) and prevalence. Additionally, we will demonstrate that certain classes of spatial models (with diffusion terms) have only real spectrum.

John D. Reeve
Dept. of Zoology
Southern Illinois University Carbondale
Carbondale, IL 62901

Title: Testing traveling wave theory using a microbial predator-prey system

There are a number ecological phenomena that display traveling waves, including the spread of invasive species, natural enemies, and diseases. An extensive theory has been developed to explain these phenomena, but has not been rigorously tested. One reason is that dispersal processes are difficult to quantify in the field. Another is the abstract nature of dispersal in the models, with only a tenuous connection to individual behavior. Both of these problems can be addressed using a laboratory system, consisting of Dictyostelium amoebas and their bacterial prey. This system shows a broad range of wave behaviors, population growth rates, and varying types of dispersal, thereby providing many test cases for the theory. One insight already provided by this system is the nature of the dispersal process. Most theory assumes that dispersal is diffusive in nature, but movement in some Dictyostelium species may be better described by the telegraph equation. The discontinuous wavefronts seen in some Dictyostelium species may be examples of wavefronts generated by this dispersal process.

Liblin Rong
Oakland University

Title - Multistage models in HIV infection and treatment

Abstract
HIV infection and replication involves multiple intracellular processes. Different classes of antiretroviral drugs target different stages. Some studies showed that patients receiving the integrase inhibitor raltegravir based therapy were faster to achieve undetectable viral load than other therapy and that treatment intensification with raltegravir led to a lower viral load and an increase in 2-LTR, a marker for ongoing viral replication. In this talk, using multistage models we will provide a quantitative and systematic comparison of the effect of different drug classes on HIV decay dynamics and particularly explain the viral load decline in HIV patients treated with raltegravir-based regimens. We will also evaluate the influence of raltegravir intensification on viral load and 2-LTR dynamics in HIV patients on suppressive antiretroviral therapy.

Simplifications of the Principal Minor Assignment Problem

Minerva Catral
Department of Mathematics
Xavier University

Given a vector $\mathbf{v}$ in $\mathbb{R}^{2^n}$, the *principal minor assignment problem* asks whether there is an $n \times n$ matrix with its 2^n principal minors given by $\mathbf{v}$. A simplified form of the problem was introduced by Brualdi, Deaett, Olesky and van den Driessche using the notion of principal rank characteristic sequences. The *principal rank characteristic sequence* of an $n \times n$ matrix is a sequence $r_0]r_1 \ldots r_n$, where $r_k \in \{0, 1\}$ such that for $k = 1, \ldots, n$, $r_k = 1$ if and only if there is a nonzero principal minor of order k, and $r_0 = 1$ if and only if there is a 0 on the main diagonal. To obtain more information, we refine this sequence by introducing an enhancement of the principal rank characteristic sequence. The *enhanced principal rank characteristic sequence* of a symmetric $n \times n$ matrix is a sequence $\ell_1 \ell_2 \ldots \ell_n$ where ℓ_k is $\mathtt{A}$, $\mathtt{S}$, or $\mathtt{N}$, according as all, some, or none of its principal minors of order k are nonzero. We discuss results on principal rank characteristic sequences and enhanced principal rank characteristic sequences for symmetric matrices over various fields, including attainability of some families of sequences and conditions that preclude attainability for some other families of sequences.

A Generalization of Young-Type Inequalities

Daeshik Choi
Department of Mathematics and Statistics
Southern Illinois University Edwardsville

The classical Young's inequality states that if $a, b > 0$ and $0 \leq v \leq 1$, then

$$(1-v)a + vb \geq a^{1-v}b^v.$$

There are many refinements of Young's inequality and its reverse. Kittaneh and Manasrah showed

$$(1-v)a + vb \geq a^{1-v}b^v + r_0\left(\sqrt{a} - \sqrt{b}\right)^2,$$

$$(1-v)a + vb \leq a^{1-v}b^v + R_0\left(\sqrt{a} - \sqrt{b}\right)^2,$$

where $r_0 = \min\{v, 1-v\}$ and $R_0 = \max\{v, 1-v\}$. Hirzallah and Kittaneh, and He and Zou, respectively, obtained other refinements:

$$((1-v)a + vb)^2 \geq \left(a^{1-v}b^v\right)^2 + r_0^2(a-b)^2,$$

$$((1-v)a + vb)^2 \leq \left(a^{1-v}b^v\right)^2 + R_0^2(a-b)^2.$$

In this talk, we will present a generalization of the above inequalities using a simple but useful result. As applications, we will give some inequalities involving positive definite matrices.

A Simple Spectral Algorithm for Recovering Planted Partitions

Sam Cole
Department of Mathematics, Statistics, and Computer Science
University of Illinois Chicago

In this talk, we consider the *planted partition model*, in which $n = ks$ vertices of a random graph are partitioned into k "clusters", each of size s. Edges between vertices in the same cluster and different clusters are included with constant probability p and q, respectively (where $0 \leq q < p \leq 1$). We give an efficient algorithm that, with high probability, recovers the clustering as long as the cluster sizes are at least $\Omega\left(\sqrt{n}\right)$. Our algorithm is based on projecting the graph's adjacency matrix onto the space spanned by its largest eigenvalues and using the result to recover one cluster at a time. While certainly not the first to use the spectral approach, our algorithm is arguably the simplest to do so: there is no need to randomly partition the vertices beforehand, and hence there is no messy "cleanup" step at the end. We also use a novel application of the Cauchy integral formula to prove its correctness.

Symmetric Tensor Powers and Quantum Variables

Philip Feinsilver
Department of Mathematics
Southern Illinois University Carbondale

In this talk, it is shown how spectra are given by the Lie homomorphism corresponding to symmetric tensor powers of matrices. This gives rise to interpreting the matrices as quantum variables with the classical variables given by diagonal matrices revealing the spectra.

Tree Sign Patterns that Require $\mathbb{H}_n$

Wei Gao
Department of Mathematics and Statistics
Auburn University

A sign pattern (matrix) $\mathcal{A}$ is a matrix whose entries are from the set $\{+, -, 0\}$. The qualitative class of $\mathcal{A}$, denoted $Q(\mathcal{A})$, is defined as $Q(\mathcal{A}) = \{B \in M_n(\mathbb{R}) \mid \text{sgn}(B) = \mathcal{A}\}$. The refined inertia of a square real matrix B, denoted $\text{ri}(B)$, is the ordered 4-tuple $(n_+(B),\ n_-(B),\ n_z(B),\ 2n_p(B))$, where $n_+(B)$ (resp., $n_-(B)$) is the number of eigenvalues of B with positive (resp., negative) real part, $n_z(B)$ is the number of zero eigenvalues of B, and $2n_p(B)$ is the number of pure imaginary eigenvalues of B. For $n \geq 3$, the set of refined inertias $\mathbb{H}_n = \{(0, n, 0, 0), (0, n-2, 0, 2), (2, n-2, 0, 0)\}$ is important for the onset of Hopf bifurcation in dynamical systems. An $n \times n$ sign pattern $\mathcal{A}$ is said to require $\mathbb{H}_n$ if $\mathbb{H}_n = \{\text{ri}(B) | B \in Q(\mathcal{A})\}$. Bodine et al. conjectured that no $n \times n$ irreducible sign pattern that requires $\mathbb{H}_n$ exists for n sufficiently large, possibly $n \geq 8$.

In this talk, we discuss the star and path sign patterns that require $\mathbb{H}_n$. It is shown that for each $n \geq 5$, a star sign pattern requires $\mathbb{H}_n$ if and only if it is equivalent to one of the five sign patterns identified in the talk. This resolves the above conjecture. It is also shown that no path sign pattern of order $n \geq 5$ requires $\mathbb{H}_n$.

A New Characterization of Simultaneous Lyapunov Diagonal Stability via Hadamard Products

Mehmet Gumus
Department of Mathematics
Southern Illinois University Carbondale

The problem of simultaneous Lyapunov diagonal stability (viz. the existence of a common diagonal Lyapunov solution) on a matrix set $\mathcal{A}$ arises from the study of Lyapunov diagonal stability on a single matrix and from the area of interconnected time-varying and switched systems. In this talk, we present a new characterization for simultaneous Lyapunov diagonal stability involving Hadamard multiplications of the matrices in $\mathcal{A}$ by positive semidefinite matrices. This extends a useful characterization, due to Kraaijevanger, of Lyapunov diagonal stability in terms of the P-matrix property under similar Hadamard multiplications. Our development mainly relies on a new notion called $\mathcal{P}$-sets, which is a generalization of P-matrices, and a recent result of Berman, Goldberg, and Shorten. An application of the new characterization is also provided.

Some New Developments of Sylvester-Type Matrix Equations

Zhuo-Heng He
Department of Mathematics and Statistics
Auburn University

In this talk, we consider some systems of coupled Sylvester-type matrix equations. We give some necessary and sufficient solvability conditions for some systems of coupled Sylvester-type matrix equations in terms of ranks and generalized inverses of matrices. We also derive the general solutions to these systems when they are solvable.

Generalized Inverses of Infinite Matrices

Bhaskara Rao Kopparty
Department of Computer Information Systems
Indiana University Northwest

We develop a theory of generalized inverses of infinite matrices. We prove several results and propose several problems.

Toeplitz Matrices are Unitarily Similar to Symmetric Matrices

Jianzhen (Jason) Liu
Department of Mathematics and Statistics
Auburn University

We prove that Toeplitz matrices are unitarily similar to complex symmetric matrices. Moreover, two $n \times n$ unitary matrices that uniformly turn all $n \times n$ Toeplitz matrices via similarity to complex symmetric matrices are explicitly given, respectively. When $n \leq 3$, we prove that each complex symmetric matrix is unitarily similar to some Toeplitz matrix, but the statement is false when $n > 3$.

Numerical Computation of Bifurcations in Large Equilibrium Systems in MATLAB

Mark Pekkér (Friedman)
Department of Mathematical Sciences
University of Alabama Huntsville

Parametrized nonlinear equations are a basis for scientific and engineering problems. In these problems it is crucial to detect and classify the qualitative changes in the solution structure as the problem parameters vary. The principal approach of numerical bifurcation analysis is based on continuation of solutions to well-defined operator equations. Such computational results give a deeper understanding of the solution behavior, stability, multiplicity, and bifurcations.

1) Introduction to numerical bifurcation analysis:
   a) Basic nonlinear phenomena: examples of bifurcations.
   b) Numerical bifurcation analysis: continuation and related issues.
2) Numerical computation of bifurcations in large equilibrium systems (US-Europe collaboration):
   a) Resolving algorithmic issues: subspace reduction via the Continuation of Invariant Subspaces (CIS) algorithm.
   b) MATLAB bifurcation software Cl_matcontL.
3) Examples:
   a) Bifurcations in 1D Brusselator.
   b) Deformation of a 2D arch.
   c) Multi-stationary and oscillatory modes in a model of free radicals generation by the Mitochondrial Respiratory Chain.
   d) Bifurcations in a one-dimensional transport model of a fusion plasma.

Matrix Asymptotic Results and Their Lie Extensions

Tin-Yau Tam
Department of Mathematics and Statistics
Auburn University

We will discuss some recent asymptotic results in matrix space and their extensions in Lie group, namely, (1) Beurling-Gelfand-Yamamoto's theorem and the generalization of Huang and Tam, (2) QR and Iwasawa asymptotic results of Huang and Tam, (3) Francis-Kublanovskaya's QR algorithm and the generalization of Holmes, Huang and Tam, (4) Rutishauer's LR algorithm and the generalization of Thompson and Tam. These results are related to several important matrix decompositions, namely, SVD, QR decomposition, Gelfand-Naimark decomposition, Jordan decomposition and their counterparts, Cartan decomposition, Iwasawa decomposition, Bruhat decomposition, complete multiplicative Jordan decomposition.

Statistical Estimation and the Affine Grassmannian

Ken Sze-Wai Wong
Department of Statistics
University of Chicago

Statistical estimation problems in multivariate analysis and machine learning often seek linear relations among variables. This translates to finding an affine subspace from the sample data set that, in an appropriate sense, either best represents the data set or best separates it into components. In other words, statistical estimation problems are optimization problems on the affine Grassmannian, a noncompact smooth manifold that parameterizes all affine subspaces of a fixed dimension. The affine Grassmannian is a natural generalization of Euclidean space, points being 0-dimensional affine subspaces. The main objective of this talk is to show that, like the Euclidean space, the affine Grassmannian can serve as a concrete computational platform for data analytic problems — points on the affine Grassmannian can be concretely represented and readily manipulated; distances, metrics, probability densities, geodesics, exponential map, parallel transport, etc, all have closed form expressions that can be easily calculated; and optimization algorithms, including steepest descent, Newton, conjugate gradient, have efficient affine Grassmannian analogues that use only standard numerical linear algebra.

Fast and Superfast Eigenvalue Solutions for Structured Matrices

Jianlin Xia
Department of Mathematics/Department of Computer Science
Purdue University

We consider the eigenvalue solution of some large structured matrices, including banded matrices and matrices with small off-diagonal numerical ranks. When the matrices are Hermitian, we present a superfast (nearly $O(n)$ complexity) divide-and-conquer algorithm for finding all the eigenvalues as well as all the eigenvectors (in a structured form). The matrices can be represented or approximated by certain rank structured forms. We show how to preserve the rank structure throughout the dividing process after recursive updates, and how to quickly perform stable eigendecompositions of the structured forms. We further discuss the structured perturbation analysis, i.e., how the compression of the off-diagonal blocks impacts the accuracy of the eigenvalues.

For non-Hermitian cases, we design a fast (nearly $O(n^2)$ complexity) contour-integral eigensolver for finding selected or all the eigenpairs. The design is based on a series of analytical and computational techniques, such as the analysis of filter functions, quick and reliable eigenvalue count via low-accuracy matrix approximations, and fast shifted factorization update. We show that a filter function based on the Trapezoidal rule has nearly optimal decay in the complex plane away from the unit circle.

The algorithm and analysis are useful for finding the eigendecompositions of matrices arising from some important applications, such as banded matrices, Toeplitz matrices, and certain sparse or dense discretized problems. The methods can also benefit many other computations such as SVDs and PDE solutions. This is joint work with James Vogel, Xin Ye, and Raymond Chan.

Remarks on Some Inequalities Related to Positive Block Matrices

Pingping Zhang
Department of Mathematics
Chongqing University of Posts and Communications

In this talk, I will show that

$$[\min\{m,n\}\mathrm{tr}(A_{i,j})I_n - A_{i,j}]_{i,j=1}^m$$

is a block PPT matrix when $[A_{i,j}]_{i,j=1}^m \in \mathbb{M}_m(\mathbb{M}_n)$ is a positive block matrix, where $\mathbb{M}_m(\mathbb{M}_n)$ is the set of $m \times m$ block matrices with $n \times n$ matrices as entries. Some related inequalities are derived.

QUANTUM INFORMATION ABSTRACTS

Russell Ceballos
Southern Illinois University Carbondale

Title: G-Consistency of the 2-Qudit Incoherent Unitary Group

Abstract: In order to properly describe dynamical quantum processes of open systems, one has to be sure that the dynamical maps representing such processes are well defined. The minimal requirements needed to ensure that dynamical maps of open quantum systems are well defined has led to the introduction of G-Consistent subsets of system-environment density operators. In this talk I will present new results proving that for each Q-consistent subset of system-environment states Q, there exists a one-to-one correspondence between Tr_E(R) and R, where Q is defined to be the group of dxd coherence non-generating unitary operators (i.e. the incoherent unitary group).

Eric Chitambar
Southern Illinois University

Title: A Unified Picture of Quantum Entanglement and Classical Secret Key

Abstract: In this talk I describe intriguing similarities between the quantum theory of entanglement and the classical theory of secret key. Just as entanglement can be shared by two or more quantum systems, secret correlations can be shared by two or more classical systems, whose states are described by probability distributions. Entanglement cannot be increased under local (quantum) operations and classical communication, and likewise secret correlations cannot be increased under local (classical) operations and public communication. This work will review recent work exploring the common resource-theoretic structure of entanglement and secrecy. Joint work with Min-Hsiu Hsieh, Andreas Winter, Benjamin Fortescue, and Borzumehr Toloui

Daniel Dilley
Southern Illinois University Carbondale

Title: The Quantum Violation of CHSH Inequality

We present a new theorem pertaining to a CHSH/Bell inequality violation that focuses on measurement directions producing nonlocal correlations. We move on to show the consequences of our result and explain its importance in quantum information theory. We map the regions on the Bloch sphere that produce nonlocal correlations for four measurement directions for an arbitrary two-qubit state ρ and a maximally entangled state Ψ.

TITLE: TRADE-OFFS IN DISTINGUISHING TWO-QUBIT STATE PREPARATIONS USING ONE-WAY LOCC

By: Alvin Gonzales

Southern Illinois University Carbondale

Quantum state discrimination is a fundamental problem in quantum information science. We investigate the optimal distinguishability of orthogonal two-qubit (bipartite) quantum states. The scenario consists of three parties: Alice, Bob, and Charlie. Charlie prepares one of two orthogonal states and sends one qubit to Alice and the other to Bob. Their goal is to correctly identify which state Charlie sent. In most state discrimination scenarios, it is assumed that Alice and Bob can freely communicate with one another so as to collectively agree on the best guess.

In this research, we consider a more restricted setting where only one-way classical communication is possible from Alice to Bob. Under this setting, we study two figures of merit (i) Alice's optimal probability, P, of identifying the state , and (ii) Alice's optimal probability, $P^{\perp}$, of identifying the state along with helping Bob identify the state perfectly. We show that in general $P \neq P^{\perp}$ and we prove a theorem for when $P = P^{\perp}$. We also found that the maximum of $P - P^{\perp}$ can arbitrarily approach $1/2$.

Gilad Gour
University of Calgary

Title: Single-shot quantum resource theories

Abstract: One of the main goals of any resource theory such as entanglement, quantum thermodynamics, quantum coherence, and asymmetry, is to find necessary and sufficient conditions (NSC) that determine whether one resource can be converted to another by the set of free operations. In this talk I will present such NSC for a large class of quantum resource theories which we call affine resource theories (ARTs). ARTs include the resource theories of athermality, asymmetry, and coherence, but not entanglement. Remarkably, the NSC can be expressed as a family of inequalities between resource monotones (quantifiers) that are given in terms of the conditional min entropy. The set of free operations is taken to be (1) the maximal set (i.e. consists of all resource non-generating (RNG) quantum channels) or (2) the self-dual set of free operations (i.e. consists of all RNG maps for which the dual map is also RNG). As an example, I will discuss the applications of the results to quantum thermodynamics with Gibbs preserving operations, and several other ARTs. Finally, I will discuss the applications of these results to resource theories that are not affine.

Marius Junge
University of Illinois Urbana-Champaign

Title
Measures of asymmetry and Entropy

Abstract: We show how to apply some ideas from the theory of noncommutative spaces to obtain estimates for the measure of asymmetry for states and channels. The investigation is motivated by a paper of Spekkens and Marvian in connection with an extension of Noether's theorem. I hope to explain the connection.
Joint work with N. LaRacuente and Le Gao

Borzumehr Toloui
Southern Illinois University Carbondale

Entanglement combing and its classical counterpart

Abstract: Entanglement combing is a protocol for transforming a pure multipartite entangled state into pairs of bipartite entangled states through local operations without significant loss. Even though entanglement is an inherently quantum property, in some contexts it has an analogue in classical information theory in the form of shared classical correlation between parties and secret keys. In this talk, I describe the quantum entanglement combing protocol and a classical counterpart of it, and discuss their similarities and differences.

Xiaoting Wang
Louisiana State University
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Title: Time-optimal control via differential geometry

Abstract:
Compared with many other methods which only give time sub-optimal designs, the quantum brachistochrone equation has a great potential to provide accurate time-optimal protocols for essentially any quantum control problem. So far it has been of limited use, however, due to the inadequacy of conventional numerical methods to solve it. Here, using differential geometry, we reformulate the quantum brachistochrone curves as geodesics on the unitary group. This identification allows us to design a numerical method that can efficiently solve the brachistochrone problem by first solving a family of geodesic equations.