(9 points)

1. Evaluate each of the following. **Show all work and give a single integer as your answer**.

   a) $6! - 4! + 0!$

   b) $P(7,7)$

   c) $C(30,2)$

In problems 2,3 and 4 you may leave your answers in terms of products, powers, permutations and/or combinations. You need not simplify your answers.

(10 points)

2. A certain website requires users to log on using a security password.

   a) If passwords must consist of six letters, followed by a single digit, determine the total number of possible distinct passwords.

   b) If passwords must consist of six non-repeated letters, followed by a single digit, determine the total number of possible distinct passwords.

(15 points)

3. A probability that a certain machine turns out a defective item is 0.05. Find the probabilities that in a run of 75 items, the following results are obtained.

(a) Exactly 5 defective items

(b) No defective items

(c) At least 1 defective item.

(15 points)

4. A basket contains 7 red apples and 4 yellow apples. A sample of 3 apples is drawn. Find the probabilities that the sample contains the following.

a) All red apples

b) 2 yellow and 1 red apple

c) More red than yellow apples.

(10 points)

5. a) Find the equation of the line through the points (-6, 3) and (2, -5). Give your answer in slope-intercept form.

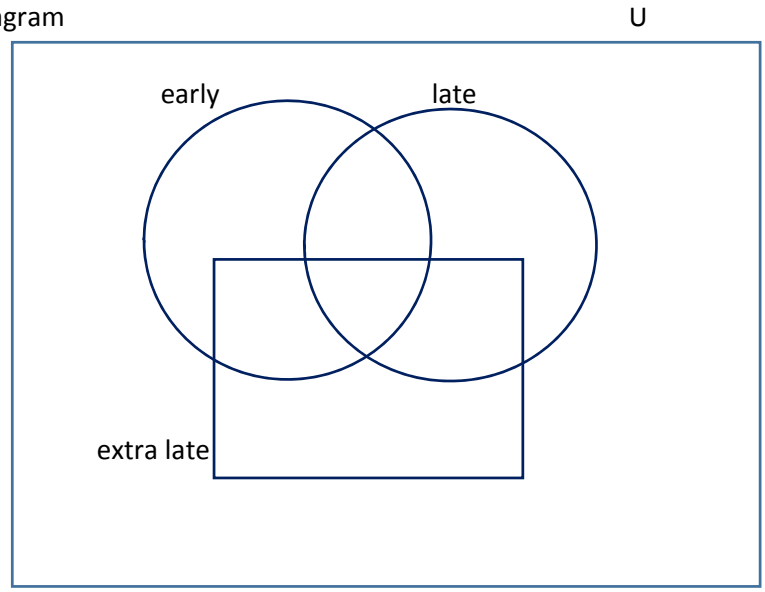
b) Are the lines with equations $4x - 2y = 9$ and $2x - y = 10$ parallel, intersecting or coincident? Justify your answer.

(12 points)

6. Toward the middle of the harvesting season, peaches for canning come in three types, early, late and extra late, depending on the expected date of ripening. During a certain week, the following data were recorded at a fruit delivery station.

34 trucks went out carrying early peaches;
61 carried late peaches;
50 carried extra late peaches;
25 carried early and late peaches;
30 carried late and extra late peaches;
8 carried early and extra late peaches;
6 carried all three peaches;
9 carried only figs (no peaches at all).

a) Complete the Venn diagram



b) How many trucks carried only late variety peaches?

c) How many trucks carried only one type of peach?

d) How many trucks (in all) went out during this week?

(9 points)

7. Find the number of distinguishable permutations of the letters in each word.

a) LITTLE

b) SUNLIGHT

(15 points)

8. Upon arrival at a hospital's emergency room, patients are categorized according to their condition as critical, serious, or stable. In the past year;

10% of the emergency room patients were critical;
30% of the emergency room patients were serious;
The rest of the emergency room patients were stable;
40% of the critical patients died;
10% of the serious patients died; and
1% of the stable patients died.

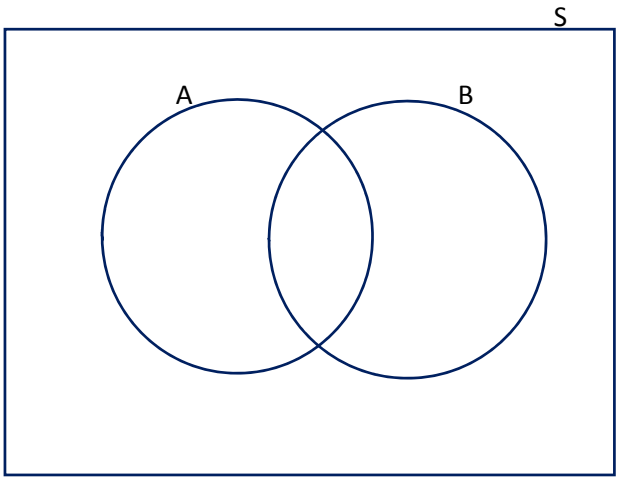
a) Construct the tree diagram for this problem, complete with the branches labeled and the assigned probabilities.

b) Given that a patient died, what is the probability that the patient was categorized as serious upon arrival?

(15 points)

9. Let A and B be events of sample space S. The $P(A \cup B) = 0.75,\ P(A) = 0.6\ , P(B) = 0.45.$

a) Complete the Venn diagram



b) Find $P(\overline{A \cup B})$.

c) Find $P(\bar{A} \cap B)$.

d) Find $P(A|B)$.

e) Are A and B independent? Justify your answer.

(15 points)

10. Given the two matrices $A = \begin{bmatrix} 5 & 2 \\ 7 & 6 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -1 & 2 \end{bmatrix}$. Find

a) AB (if possible)

b) BA (if possible)

c) Solve system of linear equations by **using the inverse of the coefficient matrix** if it exists.

$$2x + 5y = 15$$
$$x + 4y = 9$$

(15 points)

11. Solve the following system by using **reduced row-echelon** form.

$$x + y = 1$$
$$2x - z = 0$$
$$y + 2z = -2$$

(9 points)

12. The following augmented matrices represent systems of equations in terms of x, y, and z. For each find the general solution or state that no solution exists.

a) $\left[\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

b) $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

c) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$

(12 points)

13. Formulate an LP model for the following problem. DO NOT ATTEMPT TO SOLVE IT.

The Aged Wood Winery makes two white wines. Fruity and Crystal, from two kinds of grapes and sugar. One gallon of Fruity wine requires 2 bushels of Grape A, 2 bushels of Grape B, and 2 lb of sugar, and produces a profit of $12. One gallon of Crystal wine requires 1 bushels of Grape A, 3 bushels of Grape B, and 1 lb of sugar, and produces a profit of $15. The winery has available 110 of bushels of Grape A, 125 bushels of grape B, and 90 lb of sugar. How much of each wine should be made to maximize profit?

(12 points)

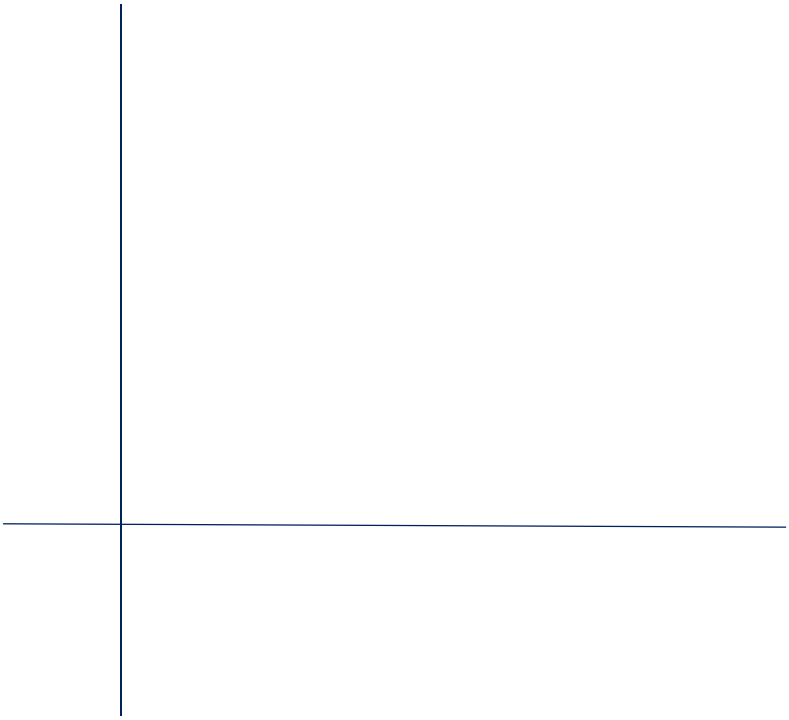
14. Solve the following linear programming problem:

Minimize $P = 5x + 3y$

Subject to the constraints

$$\begin{cases} 8x + 5y \geq 40 \\ 4x + 10y \geq 40 \\ x \geq 0, y \geq 0 \end{cases}$$

a) Sketch the feasible region



b) List the corner points of the feasible region

c) Report the complete solution.

(12 points)

15. For each tableau perform one of the following steps:

    1) If a pivot is required, write 'pivot needed', circle the pivot element, but do not pivot.
    2) If there is no optimal solution, say so and state why.
    3) If the problem is finished, report the complete solution.

a)

BV	P	x_1	x_2	x_3	s_1	s_2	s_3	RHS
s_1	0	0	3	1	1	0	1	8
s_2	0	1	2	2	0	1	0	16
s_3	0	0	0	−2	0	0	0	6
P	1	3	−4	−2	1	1	3	15

b)

BV	P	x_1	x_2	x_3	s_1	s_2	s_3	RHS
s_1	0	1	−2	5	1	0	0	10
s_2	0	1	0	2	0	1	0	12
s_3	0	0	−5	−1	0	0	1	19
P	1	2	−2	−1	0	0	0	33

c)

BV	P	x_1	x_2	s_1	s_2	s_3	RHS
x_1	0	1	0	2	1	2	6
x_2	0	0	1	2	2	4	14
P	1	0	0	4	0	5	55

(15 points)

16. Use the **simplex method** to solve the following LP problem.

Maximize $P = 25x_1 + 30x_2$

Subject to the constraints

$$\begin{cases} x_1 + x_2 \le 65 \\ 4x_1 + 5x_2 \le 300 \\ x_1 \ge 0, x_2 \ge 0 \end{cases}$$