MATH 140 **FINAL EXAM** **SPRING 2015**

Name_____ Section_____

Instructor_____

Instructions: There 15 problems on 12 pages. Please check that your copy of the exam has all the pages and all the problems numbered 1 to 15. Work in a neat and organized manner. **Show your work** on all problems. Full credit will not be given unless your work is clearly shown. On definite integrals **no credit** will be given for numerical approximations to the answer without all the supporting work.

A scientific calculator will be permitted on the final exam; however, calculators with graphic, word-processing, symbolic manipulation or programming capabilities will **not** be permitted for this exam.

Problem	Possible	Score
1	28	
2	12	
3	12	
4	6	
5	10	
6	8	
7	14	
8	10	
9	12	
10	28	
11	14	
12	12	
13	12	
14	10	
15	12	
Total	200	

(28) 1. Compute the derivative of the following functions. **Do not** simplify.

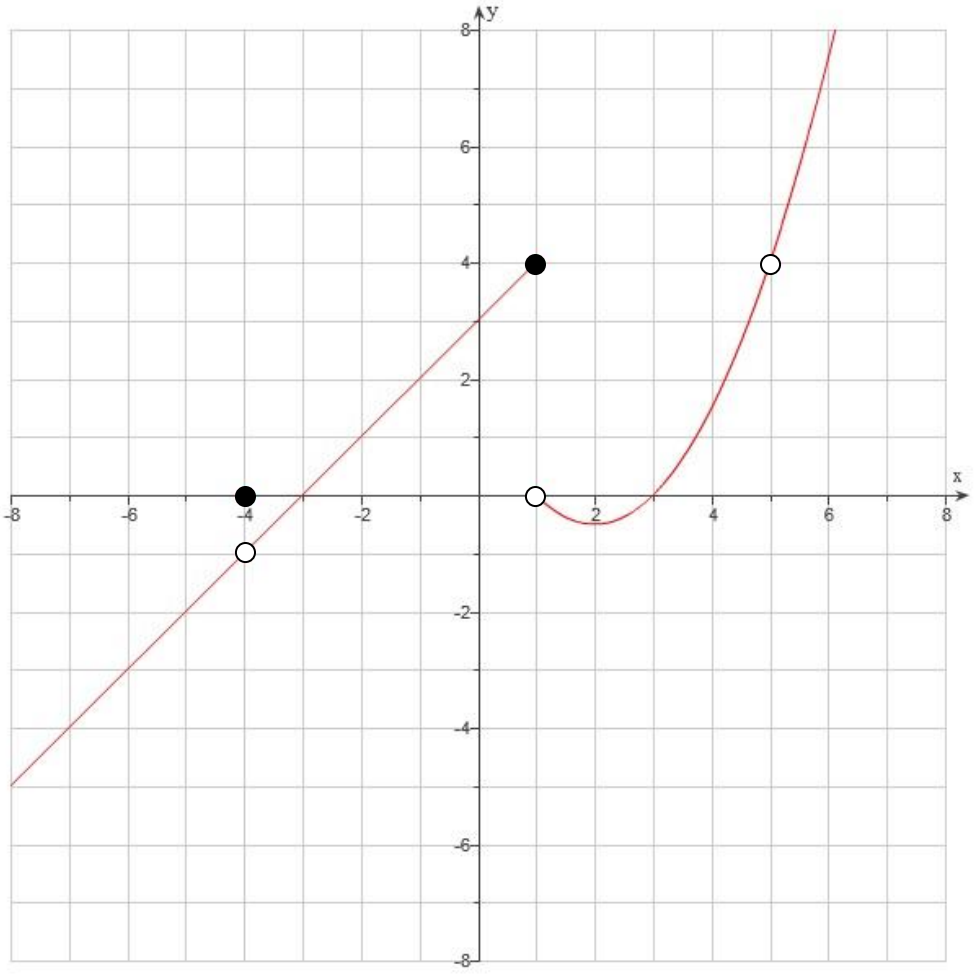
(a) $f(x) = 3\sqrt{x} + 8x^4 - 10x - \frac{3}{2}$

(b) $f(x) = x^2(e^x + 3x)$

(c) $f(x) = \frac{2x}{1 + \ln x}$

(d) $f(x) = 16\sqrt{x+9}$

(12) 2. Suppose that the graph of $y = f(x)$ is as given below. Use the graph to find the following limits. If a limit does not exist, write "DNE".



(a) $\lim_{x \to 1^+} f(x) =$

(c) $\lim_{x \to -4} f(x) =$

(b) $\lim_{x \to 1} f(x) =$

(d) Is f continuous at $x = 5$?

(12) 3. Calculate the following limits.

(a) $\lim_{x\to\infty} \dfrac{112 + x + 36x^2 + 2x^4}{-5 - 7x^2 + x^3 - 4x^4}$

(b) $\lim_{x\to -1} \dfrac{(x+1)^2}{x^2 - 1}$

c) $\lim_{x\to 1} \dfrac{\sqrt{x^2+3}}{2x-3}$

(6) 4. Let $f(x) = \dfrac{3x^2}{x^2 + x - 6}$.

a) Find all vertical asymptotes for the graph of *f*.

b) Find all horizontal asymptotes for the graph of *f*.

(10) 5. According to economic theory, the supply x of a quantity in a free market increases as the price p increases.

Suppose that the number x of Blu-Ray players a retail chain is willing to sell per week at a price of $\$p$ is given by

$$x = \frac{100p}{0.1p + 1}.$$

a) Find $\frac{dx}{dp}$.

b) Find the instantaneous rate of change of supply with respect to price when the price is \$40.

(8) 6. Find the equation of the tangent line to the graph of the function

$$y = \ln(2x - 1) \text{ at the point } (1,0).$$

(15) 7. Given the function $f(x) = x^3 + 30x^2$

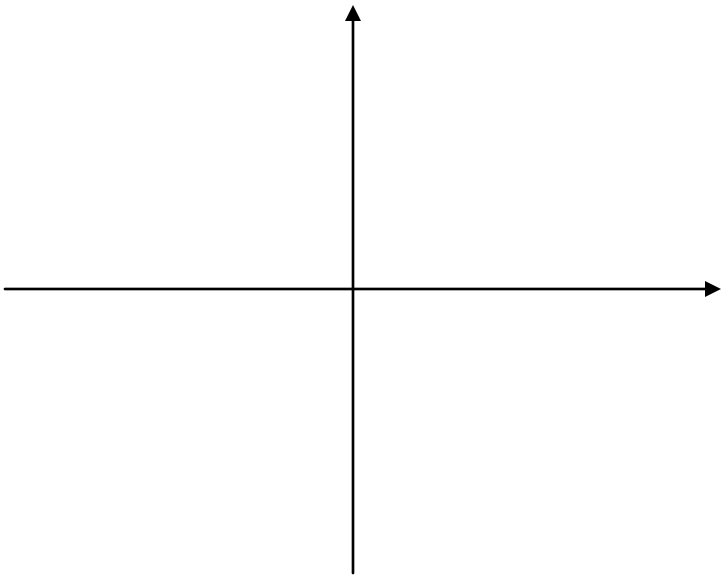
(a) Find the interval(s) on which *f* is increasing.

(b) Find all points where relative maxima and minima occur.

(c) Find the interval(s) on which *f* is concave up.

(d) Find all points of inflection.

(f) Sketch on the above axes the graph of *f(x)*. Label all relative maxima and minima and all inflection points on your graph.



(10) 8. The *total* profit (in dollars) from the sale of x cameras is

$$P(x) = 12x - 0.02x^2 - 1000$$

a) Find the *marginal* profit function.

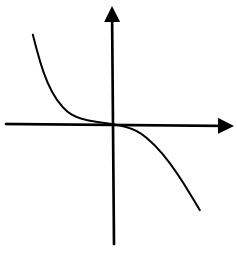
b) What is the number of cameras that the company must produce and sell in order to maximize *total* profit?

(12) 9. Match the following with the sketches of $y=f(x)$ below:

_____ (a) $f'>0$ on $(-\infty,0)$ and $f'<0$ on $(0,\infty)$
Also $f''<0$ on $(-\infty,0)$ and $f''<0$ on $(0,\infty)$

_____ (b) $f'<0$ on $(-\infty,0)$ and $f'>0$ on $(0,\infty)$
Also $f''>0$ on $(-\infty,0)$ and $f''>0$ on $(0,\infty)$

_____ (c) $f'>0$ on $(-\infty,0)$ and $f'>0$ on $(0,\infty)$
Also $f''<0$ on $(-\infty,0)$ and $f''>0$ on $(0,\infty)$



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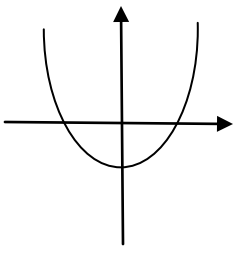

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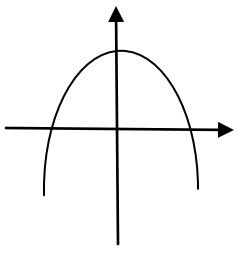

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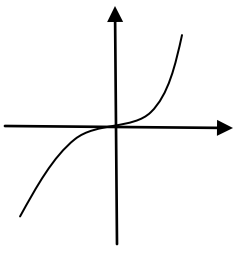

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(28) 10. Compute the following integrals.

(a) $\int\left(\frac{2}{x^3}+5x^2+1\right)dx$

(b) $\int\frac{x^3}{\sqrt{2x^4+3}}dx$

(c) $\int_1^3 te^{-t^2}dt$

(d) $\int\sqrt{x}\ln x\,dx$ (Use integration by parts.)

(16)11. Given the two functions:

$$f(x) = x^2 + 3x - 3 \text{ and } g(x) = x + 12$$

(a) Find the ordered pairs where *f* and *g* intersect.

(b) Find the area bounded by the graphs of *f* and *g*. (*Hint*: Draw a sketch first.)

(12) 12. *D(x)* is the price, in dollars per unit, that consumers are willing to pay for x units of an item, and *S(x)* is the price, in dollars per unit, that producers are willing to accept for x units.

Given $D(x) = -\frac{3}{4}x + 16$ and $S(x) = \frac{1}{2}x + 1$,

a) Find the equilibrium point.

b) Find the producer surplus at the equilibrium point.

(12)13. Let

$$f(x,y) = 3xe^{y} + 2xy^{2} + \frac{5}{3}y^{3} - 4xy + 32.$$ Find:

(a) f_y

(b) f_{yy}

(c) $f_{yy}(-4,0)$

(12) 14. Let

$$f(x,y) = 2y^{3} - 6xy - x^{2}.$$

The critical points of $f(x, y)$ are (0, 0), (9, -3). Identify each critical point as a relative minimum, a relative maximum, or a saddle point.

(12) 15. Use the method of Lagrange multipliers to find the maximum value of the function

$$f(a,b) = 25 - a^2 - b^2$$

subject to the constraint

$$2a + b = 10.$$