

Name: _____

MATH 150: Final Exam December 13, 2022

PLEASE TURN OFF ALL ELECTRONIC DEVICES. THE USE OF NOTES OR YOUR TEXTBOOK IS NOT PERMITTED DURING THIS EXAMINATION.

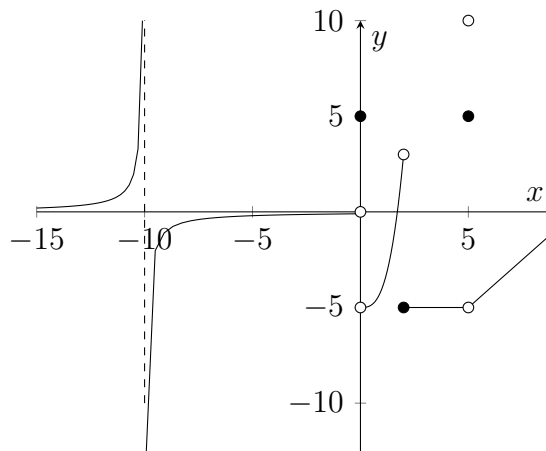
Be sure to answer all of the questions. Please check that your copy of this exam contains 12 pages with 14 questions. These 14 questions are worth a total of 200 points. All work must be shown - credit will not be given for an answer not supported by work.

Question	Points	Score
1	18	
2	18	
3	10	
4	30	
5	10	
6	7	
7	7	
8	20	
9	10	
10	8	
11	10	
12	30	
13	7	
14	15	
Total:	200	

On my honor, I have neither given nor received any aid on this examination. Furthermore, I have abided by all other aspects of the Southern Illinois University honor code with respect to this examination.

Signature: _____

1. (18 points) [2 points each] Use the graph of f below to state the value of each quantity, if it exists.



- (a) $\lim_{x \rightarrow -10^-} f(x)$
- (b) $\lim_{x \rightarrow -10^+} f(x)$
- (c) $\lim_{x \rightarrow -10} f(x)$
- (d) $f(0)$
- (e) $\lim_{x \rightarrow 0^-} f(x)$
- (f) $\lim_{x \rightarrow 0^+} f(x)$
- (g) $\lim_{x \rightarrow 5^-} f(x)$
- (h) $\lim_{x \rightarrow 5^+} f(x)$
- (i) $\lim_{x \rightarrow 5} f(x)$

2. Find the limit.

(a) (6 points) $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$

(b) (6 points) $\lim_{x \rightarrow 0} \frac{\sin(3x)}{\sin(7x)}$

(c) (6 points) $\lim_{x \rightarrow \infty} \frac{x^2}{\sqrt{2x^4 - 1}}$

3. (10 points) Use the **limit definition of derivative** to find the derivative of the function $f(x) = \sqrt{x}$.

4. Compute the derivative of each of the following functions.

(a) (6 points) $f(x) = \frac{x^4 + 7x\sqrt{x}}{\sqrt{x}}$

(b) (6 points) $f(x) = \frac{x + 1}{x^8 + 2}$

(c) (6 points) $F(x) = \int_4^{x^2} \sec(t) \tan(t) dt$

(d) (6 points) $g(x) = xe^{3x} + \ln(x^2) - \sin^{-1}(x) + 5$

(e) (6 points) $g(x) = x^{\cos(x)}$

5. (10 points) Two cars start moving from the same point at the same time. One travels south at 40 mi/h and the other travels west at 30 mi/h. At what rate is the distance between the cars increasing four hours later?

6. (7 points) Find an equation of the tangent line to the graph of $f(x) = \sin(x) \cos(x)$ at $x = 0$.

7. (7 points) If $\sin(4xy) = 3 - 3y^2$, then find $\frac{dy}{dx}$.

8. (20 points) Find the following information about the function $f(x) = \frac{x^2 - 9}{x^2 + 9}$.

$$f'(x) = \frac{36x}{(x^2 + 9)^2} \quad \text{and} \quad f''(x) = -\frac{108(x^2 - 3)}{(x^2 + 9)^3}.$$

Domain:
Horizontal Asymptote(s):
Vertical Asymptote(s):
Increasing:
Decreasing:
Local Maximum(s):
Local Minimum(s):
Concave Up:
Concave Down:
Inflection Point(s):

9. (10 points) Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

10. (8 points) Find the absolute maximum and absolute minimum of values f on the given interval. (Make sure to give exact answers.)

$$f(x) = x\sqrt{4 - x^2}, \quad [-1, 2]$$

11. (10 points) Assume that a particle moves in a straight line and has acceleration function

$$a(t) = 1 - 6t + 48t^2.$$

If the initial velocity is 2 ft/sec and the initial position is 1 foot, find its position function $s(t)$.

12. Evaluate each integral.

(a) (6 points) $\int (\sec^2(3x) + e^x + x^3 + 3^x) \, dx$

(b) (6 points) $\int_0^2 (3x + 1) \, dx$

(c) (6 points) $\int \left(\frac{4 + 6x + x^2}{\sqrt{x}} \right) dx$

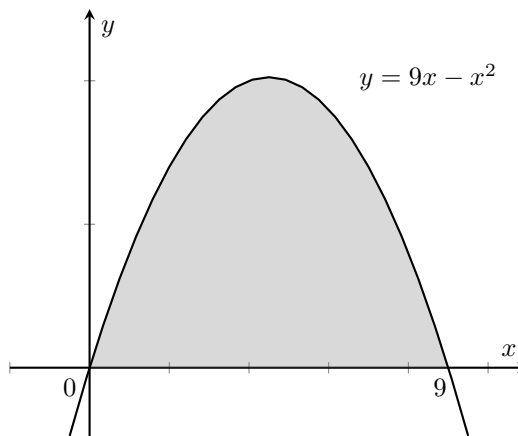
(d) (6 points) $\int x\sqrt{3x-1} dx$

(e) (6 points) $\int_e^{e^4} \frac{1}{x\sqrt{\ln(x)}} dx$

13. (7 points) Find the area of the region enclosed by the graphs of $y = \cos(x)$ and $y = \sin(x)$ on $[0, \frac{\pi}{4}]$. (Make sure to give an exact answer.)

14. Set up, but **do not evaluate**, an integral that represents the volume of the solid generated by revolving the region bounded by $f(x) = 9x - x^2$, the x -axis, $x = 0$ and $x = 9$ about:

(a) (5 points) the x -axis.



(b) (5 points) the y -axis.

(c) (5 points) the line $y = -3$.