Sample Final Exam 2 For MATH 150

You are responsible for all material covered in the course syllabus. These Sample Finals are meant to give students a general feel for the length, scope and difficultly of a Final Exam. The best way to use these Sample Finals is to study all the course material, including homeworks, worksheets, quizzes, tests and the textbook thoroughly. Then you can take a Sample Final in two hours without using the textbook, notes or any thing not permitted during the actual final. Then compare your answers with those provided. Then find your mistakes and go back and study the sections of the textbook you are still weak on. Then repeat.

Study hard and good luck!

1. (20 points) Find the following limits. Justify your answers.

a. $\lim_{\theta \to \pi} \cos(\theta + \sin\theta)$

b. $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$

c. $\lim_{y \to -1^+} \sin^{-1} y$

d. $\lim_{t \to 0} \dfrac{|t|}{t}$

2. (10 points) Let $g(x) = \begin{cases} \frac{1}{x} & \text{if} \quad x < 0, \\ 2x & \text{if} \quad 0 \le x < 1, \\ x^2 & \text{if} \quad x \ge 1. \end{cases}$

a. Find the following limits. (1 point each)

i. $\lim_{x \to 0^-} g(x)$

ii. $\lim_{x \to 0^+} g(x)$

iii. $\lim_{x \to 1^-} g(x)$

iv. $\lim_{x \to 1^+} g(x)$

v. $\lim_{x \to -\infty} g(x)$

b. Sketch the graph of $y = g(x)$.

3. (10 points) Find the following limits. Justify your answers.

a. $\displaystyle \lim_{\phi \to \left(-\frac{\pi}{2}\right)^-} \sec\phi$ b. $\displaystyle \lim_{t\to\infty} \frac{t+2}{\sqrt{9t^2+1}}$

4. (10 points) Derive the formula for the derivative of the inverse tangent function.

$$\left(\tan^{-1} x\right)' = \frac{1}{1+x^2}.$$

5. (30 points) Find the derivative of each function below. You need not simplify your answers.

   a. $4x^3 + x\sin(2x) + \tan(3x^2)$

   b. $\frac{1}{\sqrt{7x^2+1}}$

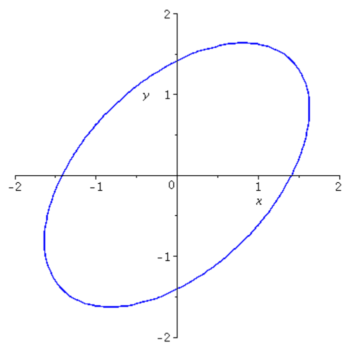
   c. $\ln\cos x$

   d. $\frac{x^2}{1+x^3}$

   e. $e^{\sec 3t}$

   f. $\sin^2\theta + \cos^2\theta$

6. (10 points) The graph of $x^2 - xy + y^2 = 2$ is the ellipse shown below. Find the equation of the tangent line that meets the ellipse at the point $(0, \sqrt{2})$. Express your answer in slope-intercept form. Draw it on the graph.



7. (15 points) a. State the Mean Value Theorem by filling in the blank spaces below.

The Mean Value Theorem: Let $[a, b]$ be a closed bounded interval. Suppose f is a function whose domain contains $[a, b]$ that satisfies the following two conditions.

1. f is ______________________ on $[a, b]$.
2. f is ______________________ on (a, b).

Then there exists a number $c \in (a, b)$ such that $f'(c) =$ ____________ .

b. Draw a picture that illustrates the idea behind the Mean Value Theorem.

c. Let $f(x) = 3x - 1 - \cos x$. Show that $f(x) = 0$ has exactly one real solution. Hint: first use the Intermediate Value Theorem to show there is a solution in the interval $(0, \pi)$.

8. (10 points) Find two positive numbers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

9. (10 points) Sketch the graph of $y = e^{-x^2}$. Find the x coordinates of the two inflection points. Indicate where the graph is concave up and concave down.

10. (10 points) Sketch the graph of $y = \dfrac{x}{x^2+1}$. Find the values and locations of the absolute maximum and absolute minimum.

11. (10 points) A projectile is launched directly upward and lands 10 seconds later. What was its initial velocity? (The downward acceleration due to gravity is 32 feet/sec/sec. Ignore air resistance.)

12. (5 points) When two resistors with resistances R_1 and R_2 are connected in paraellel, then the total resistance, messured in ohms (Ω), can be found from

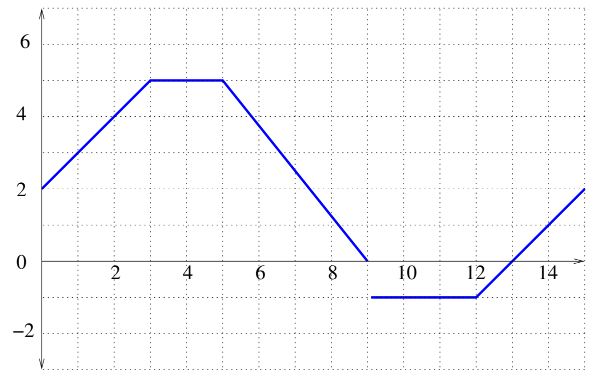
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

If R_1 and R_2 are increasing at rates of $2.0\ \Omega$ / second and $3.0\ \Omega$ / second, what is the rate of change of R when $R_1 = 10\,\Omega$ and $R_2 = 15\,\Omega$?

13. (10 points) Sketch the graph of a function $y = f(x)$ that has the following properties. You may assume f is continuous and has continuous first and second derivatives.

$f(0) = 3$.
For x in $(-\infty, -2)$, $f'(x) > 0$.
For x in $(-2, 3)$, $f'(x) < 0$.
For x in $(3, \infty)$, $f'(x) > 0$.
For x in $(-\infty, -4)$, $f''(x) > 0$.
For x in $(-4, 1)$, $f''(x) < 0$.
For x in $(1, \infty)$, $f''(x) > 0$.

14. (6 points) The graph of $y = f(x)$ is shown below. Use it to evaluate the following definite integrals. Each grid cell is 1×1.



a. $\int_0^5 f(x)\,dx$

b. $\int_5^{12} f(x)\,dx$

c. $\int_{12}^{15} f(x)\,dx$

15. (4 points) Let $g(x) = \displaystyle\int_0^{x^2} \frac{\sin t}{t}\,dt$. Find $g'(x)$.

16. (20 points) Find the following indefinite integrals.

a. $\displaystyle\int \sqrt{3x+1}\,dx$

b. $\displaystyle\int \frac{\ln x}{x}\,dx$

c. $\displaystyle\int \csc^2(3t)\,dt$

d. $\displaystyle\int \frac{x+2}{\sqrt{x^2+4x}}\,dx$

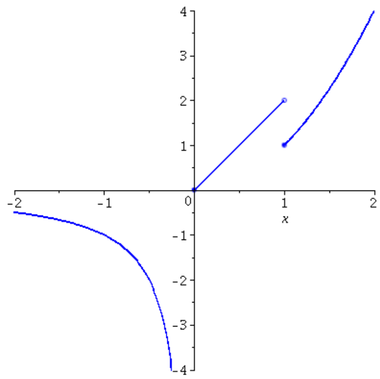
17. (10 points) Evaluate the following definite integrals.

a. $\displaystyle\int_0^2 \sqrt{4-x^2}\,dx$

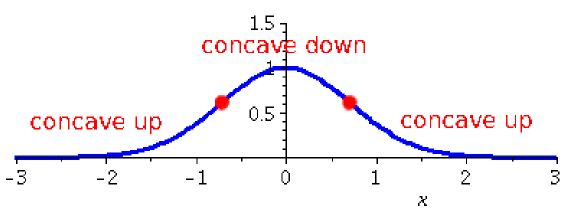
b. $\displaystyle\int_0^1 x(x^2+1)^3\,dx$

Answers

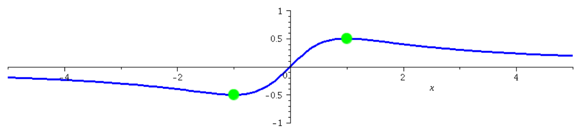
1. a. -1 b. 5 c. $-\pi/2$ d. Does not exist.
2. a. $-\infty$, 0, 2, 1, 0. b. See graph below.



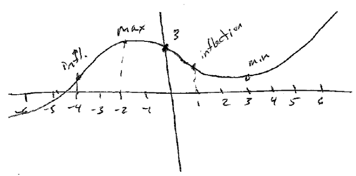
3. a. $-\infty$ (Exercise 17, Section 1.6). b. $1/3$ (Exercise 22, Section 1.6)
4. See Formula 9 in Section 3.5.
5. a. $12x^2+\sin 2x+2x\cos 2x+6x\sec 3x^2$ b. $\frac{-7x}{(\sqrt{7x^2+1})^3}$ c. $-\tan x$ d. $\frac{2x-x^4}{(1+x^3)^2}$
   e. $3e^{\sec 3t}\sec 3t\tan 3t$ f. 0
6. $y=\frac{1}{2}x+\sqrt{2}$ (Use implicit differentiation to get y', which gives you the slope.)
7. See Section 4.2.
8. $x=500$, $y=125$
9. $x=\pm 1/\sqrt{2}$. The graph is a bell curve. See below.



10. Maximum is $\frac{1}{2}$ when $x=1$. Minimum is $-\frac{1}{2}$ when $x=-1$.



11. 160 ft/sec
12. $2.4\,\Omega$/sec. See Exercise 31 in Section 2.7.
13. See graph below. This answer is not unique.



14. a. 20.5 b. 7 c. 1.5 (Compare to Exercise 29 in Section 5.2.)
15. $\frac{2\sin x^2}{x}$
16. a. $\frac{2}{9}(3x+1)^{\frac{3}{2}}+C$ b. $(\ln x)^2+C$ c. $\frac{1}{3}\cot(3t)+C$ d. $\sqrt{x^2+4x}+C$
17. a. 2π (Use geometry. See Example 3, Section 5.2) b. 15/8