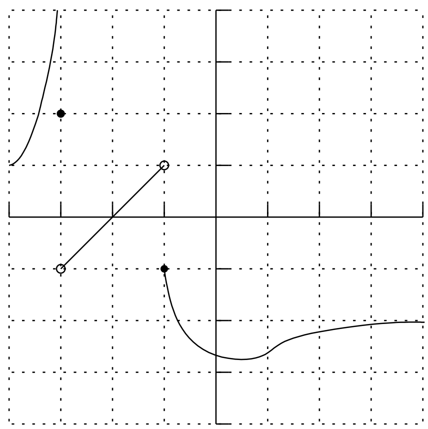
Sample Final Exam 3 For MATH 150

You are responsible for all material covered in the course syllabus. These Sample Finals are meant to give students a general feel for the length, scope and difficultly of a Final Exam. The best way to use these Sample Finals is to study all the course material, including homeworks, worksheets, quizzes, tests and the textbook thoroughly. Then work through a Sample Final in two hours without using the textbook, notes or any thing not permitted during the actual final. Then compare your answers with those provided. Then find your mistakes and go back and study the sections of the textbook you are still weak on. Then repeat.

Study hard and good luck!

1. (5 points) Study the graph of $y = f(x)$ below and answer the following limits. Each tick mark is one unit.



a. $\lim_{x \to -3^-} f(x)$

b. $\lim_{x \to -3^+} f(x)$

c. $\lim_{x \to -1^-} f(x)$

d. $\lim_{x \to -1^+} f(x)$

e. $\lim_{x \to \infty} f(x)$

2. (30 points) Evaluate the following limits when they exist. Justify your answers.

a. $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$

b. $\lim_{t \to 0} \dfrac{3 - \sqrt{t^2 - 9}}{t^2}$

c. $\lim_{x \to 0^+} x \ln x$

d. $\lim_{x \to 2} \dfrac{x - 2}{x^3 - 8}$

e. $\lim_{x \to \infty} \dfrac{x^3 + 5x}{5x^3 + x^2 + 7}$

f. $\lim_{\theta \to \left(\frac{\pi}{2}\right)^+} \tan\theta$

3. (5 points) Let $f(x) = \frac{\cot x}{x-2}$. Give all the values of x where $f(x)$ is not continuous.

4. (10 points) Find the derivative of $f(x) = \frac{1}{3x+2}$ using the definition of the derivative and the properties of limits. Show all steps.

5. (30 points) Find the following derivatives. You do not need to simplify your answers.

a. $\ln(\ln x) + \tan x^3$

b. $2^x + x \csc x^2$

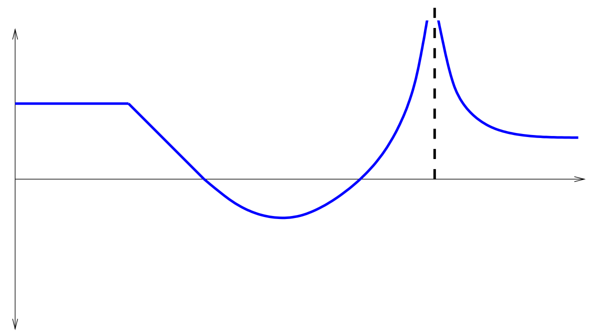
c. $\frac{\cos t}{1+\sin t}$

d. $17 + e^{\sin x^2}$

a. $x^4(1 + \sec^2 x)$

b. $6x^2 - \arctan 2x$

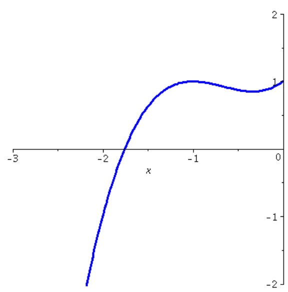
6. (10 points) The graph of a function $f(x)$ is given below. Sketch of rough graph of its derivative, $f'(x)$. (Your graph will have discontinuities.)



7. (10 points) A spherical balloon is being filled with air at a steady rate of 4 cc/min (cc = cubic centimeters). What is the rate of change of its surface area when its radius is 3 cm?

8. (10 points) Find the absolute maximum and minimum values of $f(x) = e^{3x} - 4x$ on $[-1, 2]$. Give exact answers and numerical approximations to two decimal places.

9. (5 points) Let $f(x) = x^3 + 2x^2 + x + 1$. Find the tangent line to the graph of $y = f(x)$ when $x = -2$. Express your answer in slope-intercept form. Where does this tangent line cross the x-axis? Draw the tangent line on the graph of $y = f(x)$ below. This is sometimes used to estimate where a function is zero.



10. (10 points) Suppose that $f(0) = -3$ and $f'(x) \leq -5$ for all values of x. How large can $f(2)$ possibly be? Hint: Think about the Mean Value Theorem.

11. (20 points) Let $f(x) = \dfrac{x^2 - 9}{x^2 + 9}$. Because we are nice we are going to give you that

$$f'(x) = \frac{36x}{(x^2+9)^2} \qquad \text{and} \qquad f''(x) = \frac{-108(x^2-3)}{(x^2+9)^3}.$$

(a-g are 10 points in total, h is 5 points.)

a. Where is $f(x) = 0$? Where is the y-intercept? Mark these on the axes below.

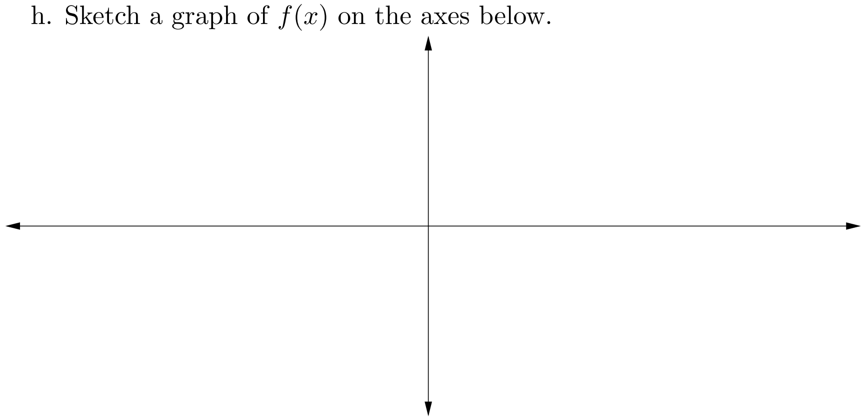
b. What is the asymptotic behavior of $f(x)$? That is, what value does it approach as $x \to \pm\infty$? Draw any horizontal asymptotes on the axes below.

c. Find any vertical asymptotes or state that there are none.

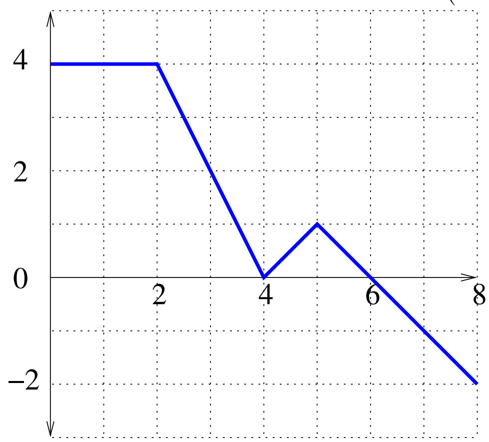
d. For which values of x is $f(x)$ increasing? e. For which values of x is $f(x)$ decreasing?

f. On which intervals is $f(x)$ concave up? ġ. On which intervals is $f(x)$ concave down?

h. Sketch a graph of $f(x)$ on the axes below.



12. (15 points) Let $g(x) = \int_0^x f(t)\,dt$, where $f(t)$ is the function whose graph is shown below. (a-e are 2 points each, f is 5 points.)



a. Evaluate $g(8)$.

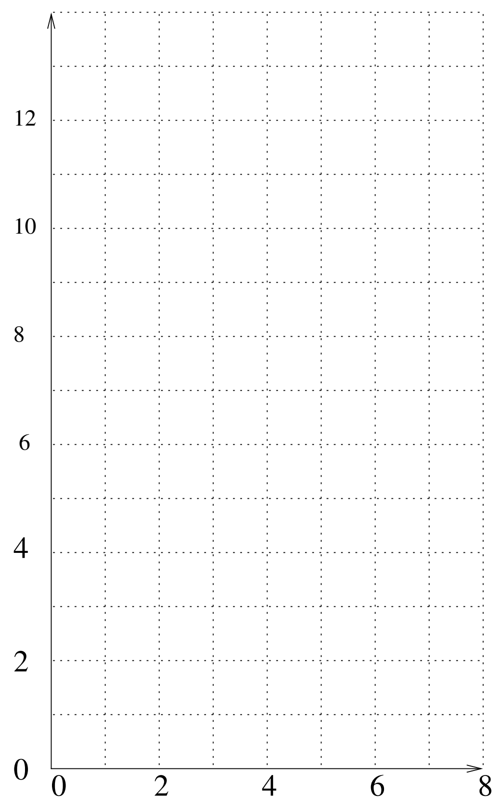
b. On what interval is $g(x)$ increasing?

c. On what interval is $g(x)$ decreasing?

d. For what value of x does $g(x)$ have a local maximum?

e. On $(2, 4)$ is $g(x)$ concave up or concave down?

f. Sketch a rough graph of $g(x)$.



13. (5 points) Express the limit below as a definite integral on $[0, \pi]$. Do not try to evaluate it. (Hint: Recall the definition of the integral as a limit of Riemann sums.)

$$\lim_{n\to\infty} \sum_{i=1}^{n} x_i \sin(x_i)\, \Delta x$$

14. (25 points) Evaluate the following integrals.

a. $\displaystyle\int \frac{\sec^2(\ln x)}{x}\, dx$

b. $\displaystyle\int \tan x \ln(\cos x)\, dx$

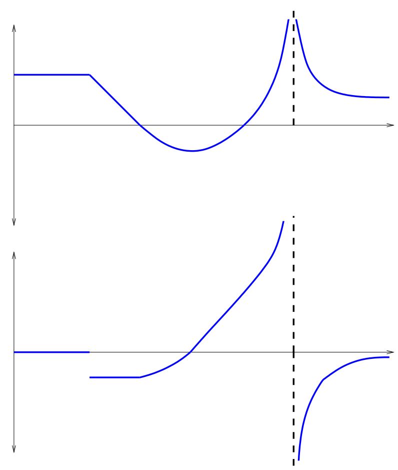
c. $\displaystyle\int \cos 2x + \frac{x}{x+1} + xe^{x^2}\, dx$

d. $\displaystyle\int_e^{e^3} \frac{1}{x}\, dx$

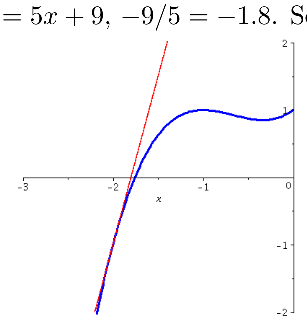
e. $\displaystyle\int_{-2}^{2} \frac{x^3 + x\cos x}{x^2+1} + x^2\, dx$

Answers

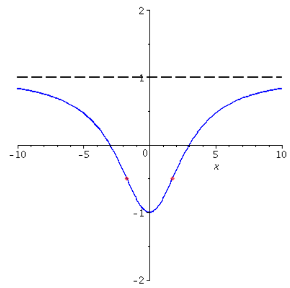
1. a. ∞ b. -1 c. 1 d. -1 e. -2
2. a. 0 (See Example 9, Section 1.4) b. $-1/6$ (See Example 4, Section 1.4) c. 0 (Use L'Hospital's Rule) d. $1/2$ (Factor the denominator or use L'Hospital's Rule) e. $1/5$ (You could use L'Hospital's Rule, but it will take longer.) f. ∞
3. The function is not continuous, in fact it is not even defined, for $x = 2$ and $x = n\pi$ for all integers n.
4. $\frac{-3}{(3x+2)^2}$, but you have to show the steps as in Example 4, Section 2.2.
5. a. $\frac{1}{x \ln x} + 3x^2 \sec^2 x^3$ b. $(\ln 2)2^x + \csc x^2 - 2x^2 \csc x^2 \cot x^2$, c. $\frac{-1}{1+\sin x}$ (when simplified) d. $2x \cos x^2 e^{\sin x^2}$ e. $4x^3(1 + \sec^2 x) + 2x^4 \sec^2 x \tan x$ f. $12x - \frac{2}{1+4x^2}$
6. See below.



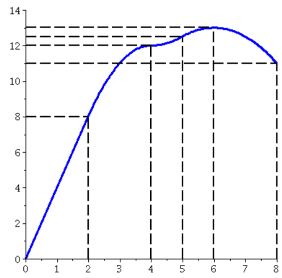
7. $8/3$ (cm)^2 / min.
8. Max is at 2 is $e^6 - 8 \approx 395.43$. Min is at $(\ln 4/3)/3$ is $4(1 - \ln 4/3)/3 \approx 0.95$.
9. $y = 5x + 9$, $-9/5 = -1.8$. See graph below.



10. 7. See Example 5, Section 4.2.
11. a. $f(\pm 3) = 0$, $f(0) = -1$ b. $y = 1$ is a horizontal asymptote for $x \to \infty$ and $x \to -\infty$. c. No. d. Increasing for $x > 0$. Decreasing for $x < 0$. e. Concave up on $(-\sqrt{3}, \sqrt{3})$. Concave down on $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$. f. See below. (Inflection points are mark with red dots.)



12. a. 11 b. $(0, 6)$ c. $(6, 8)$ d. 6 e. It is concave down. Since the slope of f is -2 the second derivative of g is -2. f. See below. See also Exercises 1 and 2 in Section 5.4.



13. $\int_0^\pi x \sin x \, dx$. See Example 1, Section 5.2.
14. a. $\tan(\ln x) + C$ b. $-(\ln \cos x)^2 + C$ c. $\frac{1}{2} \sin 2x + x - \ln |x + 1| + \frac{1}{2} e^{x^2} + C$ d. 2 e. 16/3 (Show the first part of the integrand is an odd function.)