

[50] 1. Evaluate the following integrals.

a)  $\int (\pi x - 2015) \sin(ex) dx$

b)  $\int \frac{2015x + \pi}{\pi x - x^2} dx$

c)  $\int_0^1 \frac{dx}{\sqrt{1+x^2}}$

d)  $\int_0^{\pi/2} \sin^{2\pi+1}(\theta) \cos^3 \theta d\theta$

e)  $\int \sqrt{x} \ln x dx$

- [20] 2. Evaluate the following improper integrals or conclude that they diverge. Explain your answer in each case.

a) 
$$\int_{-\pi}^0 \frac{dx}{(x + \pi)^3}$$

b) 
$$\int_{-\infty}^{+\infty} \frac{dx}{1 + (x + 2)^2}$$

[20] 3. Find each of the following limits.

a)  $\lim_{x \rightarrow 0^+} (1 + \sin 2x)^{\cot 3x}$

b)  $\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{\sin(2x)}$

- [10] 4. Find the slope of the tangent line to the curve  $x = r + \cos(t)$ ,  $y = r \tan(t) + \sin(t)$  at the point  $t = \pi/4$ , where  $r$  is any real constant.

- [10] 5. Find the arclength of the curve  $x = t$ ,  $y = t^{3/2}$  over the interval  $0 \leq t \leq 1$ .

- [20] 6. Find the area of the region that lies inside the curve  $r = 3 \sin(\theta)$  and outside the cardioid  $r = 1 + \sin(\theta)$ . (Compute the intersection points first.)

[10] 7. Find the interval of convergence of the power series.

a) 
$$\sum_{n=1}^{\infty} \frac{(2015x - 1)^n}{(n + 2)\pi^n}$$

b) Find the sum of the series 
$$\sum_{n=2}^{\infty} \left(\frac{e}{\pi}\right)^{n-1}.$$

[15] 8. Find the Maclaurin series for

a)  $f(x) = \ln(\pi + x)$

b)  $f(x) = \sin(\pi x)$

[15] 9. Use Maclaurin series to find  $\int_0^{.3} \cos(x^2) dx$  to within  $10^{-4}$ .



- [30] 10. Determine whether the following series converge absolutely, converge conditionally or diverge. Explain your answer in each case.

a) 
$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^e \ln(n^{10})}$$

b) 
$$\sum_{n=0}^{\infty} \left( \frac{(-2)^n}{n!} \right)$$

c) 
$$\sum_{k=1}^{\infty} \left( \frac{2k+1}{3k-1} \right)^k$$