[50] 1. Evaluate the following integrals.

a) $\int 3x^2 \ln(2x)\, dx$

b) $\int_0^{\pi/4} \sec^2 x \tan^3 x\, dx$

c) $\int \frac{dx}{x^2 + 6x + 25}$

d) $\displaystyle\int \frac{dx}{x(x^2+1)}$

e) $\displaystyle\int \frac{dx}{(1-x^2)^{3/2}}$

[20] 2. Evaluate the integrals or conclude that they diverge. Please explain.

a) $\displaystyle\int_{-1}^{1} \frac{dx}{x^4}$

b) $\displaystyle\int_{2}^{\infty} xe^{-x^2}\,dx$

[15] 3. Evaluate the following limits.

a) $\lim_{x \to 0} \frac{x - \sin x}{x^3}$

b) $\lim_{x \to \infty} \left(1 - \frac{2}{x}\right)^{5x}$

[35] 4. Determine whether each series is absolutely convergent, conditionally convergent, or divergent. Name the tests that you are using and show all work.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-e)^n}{n^n}$

b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{3n+1}}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n n^3}{n!}$

d) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

e) $\sum_{n=1}^{\infty} \sin n$

[10] 5. Find the interval of convergence for the power series. Be sure to check the end points.

$$\sum_{n=1}^{\infty} \frac{(3x-4)^n}{n2^n}$$

[20] 6. Find the Maclaurin series of the function

a) $f(x) = \dfrac{e^x - 1 - x}{x^2}$

b) $f(x) = \dfrac{x^3}{1 - 2x}$

[10] 7. Find the first four terms in the Taylor series for the function $f(x) = \sin x$ about $a = \dfrac{\pi}{3}$.

[10] 8. Let $f(x) = \frac{x}{\sqrt{1}} - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + ... = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{\sqrt{n}}$. Use this series to estimate

$$\int_0^{0.1} f(x)dx$$

within an error less than 0.0001. You may leave your answer as a sum of fractions.

[10] 9. Find all the points on the curve where the tangent is horizontal or vertical. Clearly label which ones are vertical and which ones are horizontal.

$$x = t^3 - 3t, \ y = t^3 - 3t^2$$

[10] 10. Find the arc length of the curve $x(t) = \cos t + t \sin t, \ y(t) = \sin t - t \cos t, \ 0 \leq t \leq 2\pi$.

[10] 11. Find the area outside the polar curve $r = 1$ and inside the polar curve $r = 2\sin\theta$.