# MATH 109 – TOPIC 1

# RADICALS



## Introduction

Welcome ... either you have been surfing the web and took a wrong turn, or you are looking for some help with trigonometry. If you are still reading, I'll assume the latter.

Trig, as you are about to find out, is a mix of several topics (functions and graphs, equations, polar coordinates, identities, ...) However, at its core, trig is a study of right triangles. If right triangles are present, then the pythagorean theorem (and radicals) can't be far behind.

## I. Pythagorean Theorem

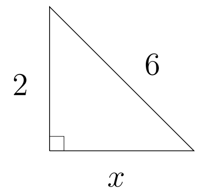
"In any right triangle, the sum of the squares of the two legs must equal the square of the hypopatemus" ... oops, I mean the hypotenuse. You probably know it better as $a^2 + b^2 = c^2$.

Here are two applications of this theorem.

**Example 1.1.** Is a triangle with sides of 5, 12, and 13 a right triangle?

Solution: Any triangle is right iff $a^2 + b^2 = c^2$. Since $5^2 + 12^2 = 25 + 144 = 169 = 13^2$, then the given triangle is a right triangle.

**Example 1.2.** Find $x$ in the given triangle.



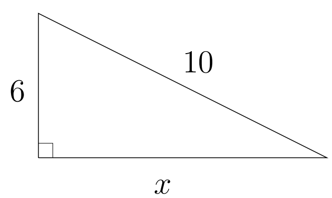

Note: The box will always be used to indicate a right angle.

Solution: With legs of 2 and $x$ and a hypotenuse of 6, the Pythagorean Theorem states:

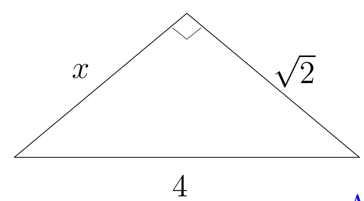
$$
\begin{aligned}
2^2 + x^2 = 6^2 \Rightarrow x &= \sqrt{6^2 - 2^2} \\
&= \sqrt{32} \\
&= 4\sqrt{2} \qquad\qquad \sqrt{ab} = \sqrt{a}\sqrt{b}
\end{aligned}
$$

**Exercise 1.** Now it's your turn. Find $x$ in the following:

a)




(b)




Answers

## II. Arithmetic with Radicals

Hopefully I've convinced you how important the Pythagorean Theorem (and square roots) will be in your study of trig. Now let's move on to radical arithmetic. Warning, this stuff is trickier (and more dangerous) than you might imagine ..., a parent consent form might be necessary before proceeding.

*Note: Math humor (much like mathematics) is an acquired taste.*

**Example 1.3.** Here are some computations involving square roots.

a) $2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$

b) $(2\sqrt{3})^2 = 4(3) = 12$

c) $\dfrac{4}{\sqrt{2}} = \dfrac{4}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{4\sqrt{2}}{2} = 2\sqrt{2}$

d) $\sqrt{\dfrac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\dfrac{1-\frac{\sqrt{3}}{2}}{2} \cdot \dfrac{2}{2}} = \dfrac{\sqrt{2-\sqrt{3}}}{2} = \dfrac{1}{2}\sqrt{2-\sqrt{3}}$

e) $\sqrt{9-x^2}$; can not be simplified

f)
$$\begin{aligned} \sqrt{(\sqrt{8x})^2 + (x-2)^2} &= \sqrt{8x + x^2 - 4x + 4} \\ &= \sqrt{x^2 + 4x + 4} \\ &= \sqrt{(x+2)^2} = |x+2| \end{aligned}$$

To see why $\sqrt{(\quad)^2} = |(\quad)|$, click here.

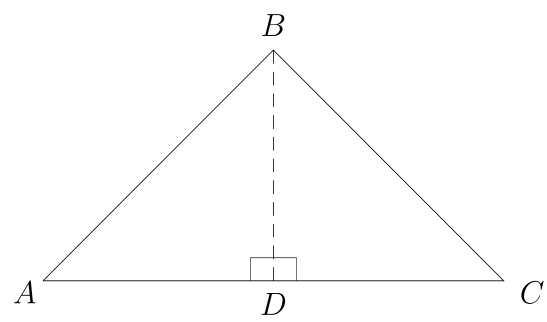
These last 2 examples are extremely important since the most common algebra errors arise with trig expressions in radicals. Here are a few more radical expressions to look over.

| | | |
|---|---|---|
| $\sqrt{4x^2} = \lvert 2x \rvert$ | $\sqrt{(2-x)^2} = \lvert 2-x \rvert$ | $\sqrt{4-x^2} \neq 2-x$ |
| $\sqrt{4\cos^2 x} = 2\lvert \cos x \rvert$ | $\sqrt{(1-\cos x)^2} = \lvert 1-\cos x \rvert$ | $\sqrt{1-\cos^2 x} \neq 1-\cos x$ |

Let's finish by trying a slightly more difficult problem:

**Exercise 2.**

Suppose you are given this figure with $\overline{AB} = 6$, $\overline{BC} = 9$ and $\overline{BD} = 5$. Could you find $AC$? Take your best shot and when you are ready to compare results, check the answer.

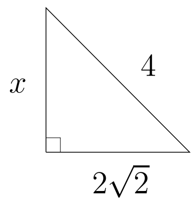



Answer

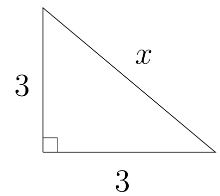
PRACTICE PROBLEMS for Topic 1 – Radicals

1.1. Find $x$ for each of the following right triangles.

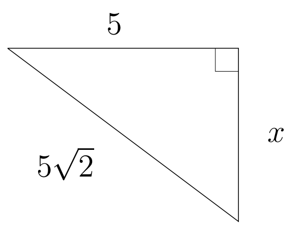
a)



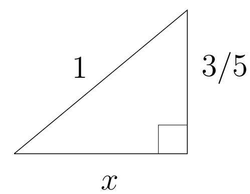

b)



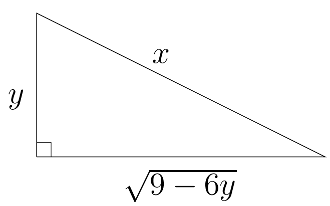

c)




d)




e)




Answers

1.2. Perform the indicated opreation.

a) $\sqrt{\dfrac{5}{4}}$

b) $\sqrt{\dfrac{4}{5}}$

c) $(2\sqrt{5})^2$

d) $(2+\sqrt{5})^2$

e) $(\sqrt{5})^6$

f) $\sqrt{\dfrac{1-\dfrac{\sqrt{2}}{2}}{2}}$

Answers

Beginning of Topic 109 Study Topics 109 Skills Assessment

Example 1.3.

f) True or False: is $\sqrt{x^2} = x$, for all values of $x$?

False. This statement is true as long as $x$ is non-negative; $\sqrt{5^2} = 5$ or $\sqrt{100^2} = 100$, and so on. But what if $x = -5$?

$$\sqrt{x^2} = \sqrt{(-5)^2} = \sqrt{25} = 5 = -x, \text{ not } x.$$

This leads to a more precise definition: $\sqrt{x^2} = |x|$. This is true for all $x$. In calculus you will be expected to use this definition.

Return to Review Topic

Exercise 1.

a) $6^2 + x^2 = 10^2$

$$x = \sqrt{100 - 36} = 8$$

b) $x^2 + \sqrt{2}^2 = 4^2$

$$x = \sqrt{16 - 2} = \sqrt{14}$$

Return to Review Topic

Exercise 2.

Use the Pythagorean theorem to find $\overline{AD}$ and $\overline{DC}$.

$$\begin{aligned}
\overline{AD} &= \sqrt{6^2 - 5^2} = \sqrt{11} \\
\overline{DC} &= \sqrt{9^2 - 5^2} = \sqrt{56} = 2\sqrt{14} \\
\overline{AC} &= \overline{AD} + \overline{DC} = \sqrt{11} + 2\sqrt{14}
\end{aligned}$$

Return to Review Topic

1.1. a) $x^2 + (2\sqrt{2})^2 = 4^2$

$x = \sqrt{16-8} = 2\sqrt{2}$

b) $x^2 = 3^2 + 3^2$

$x = \sqrt{18} = 3\sqrt{2}$

c) $x^2 + 5^2 = (5\sqrt{2})^2$

$x = \sqrt{50-25} = 5$

Did you notice in parts b) and c) that isosceles right triangles have a distinct pattern in the relationship between legs and hypotenuse?

d) $x = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

e) $x = \sqrt{y^2 + 9 - 6y} = \sqrt{(y-3)^2} = |y-3|$

Return to Problem

1.2. a) $\frac{\sqrt{5}}{2} = \frac{1}{2}\sqrt{5}$

b) $\sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

c) 20

d) $4 + 4\sqrt{5} + 5 = 9 + 4\sqrt{5}$

e) $(5^{1/2})^6 = 5^3 = 125$

f) $\sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2} \cdot \frac{2}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{1}{2}\sqrt{2-\sqrt{2}}$

Return to Problem