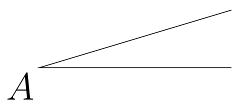
# MATH 109 – TOPIC 3

# RIGHT TRIANGLE TRIGONOMETRY

## 3a. Right Triangle Definitions of the Trigonometric Functions

In this section, we present the definitions of the trigonometric functions based on a right triangle. An alternate but equivalent method of introducing the trigonometric functions based on the unit circle will be discussed later.

Consider some acute angle, $A$.




Form a right triangle, where $C = \dfrac{\pi}{2} = 90^\circ$. (Remember that the sum of the interior angles in <u>any</u> triangle is $180^\circ$ or $\pi$ radians.)

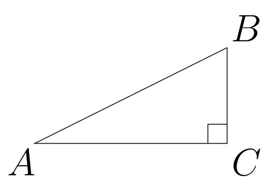



Fig. 3a.1

We have the following definitions.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AC}{AB}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin A}{\cos A} = \frac{BC}{AC}$$

$$\cot A = \frac{\text{adjacent}}{\text{opposite}} = \frac{1}{\tan A} = \frac{\cos A}{\sin A} = \frac{AC}{BC}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{1}{\cos A} = \frac{AB}{AC}$$

$$\csc A = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{\sin A} = \frac{AB}{BC}$$

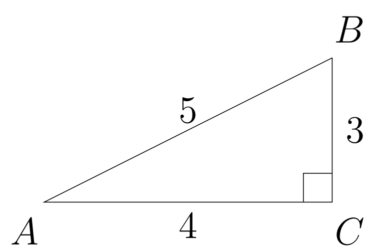
In a similiar fashion, we could start with the other acute angle in the right triangle, angle $B$, and define

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AC}{AB}, \quad \cos B = \frac{BC}{AB}, \text{ etc.}$$

Notice that opposite and adjacent are relative to the angle in question but that the definitions do not change. Also remember that since $C$ is a right angle,

$$B = 90^\circ - A^\circ = \left(\frac{\pi}{2} - A\right) \text{ radians.}$$

PRACTICE PROBLEMS for Topic 3a-Right Triangle Definitions of the Trigonometric Functions




3a.1. Find exact values for the following.

a) $\sin A =$ b) $\cos A =$

c) $\tan A =$ d) $\sin B =$

e) $\cos B =$ f) $\cot B =$

g) $\sec A =$ h) $\sin^2 A + \cos^2 A =$

i) $\csc B =$ j) $\sin\left(\frac{\pi}{2} - A\right) =$

Answers

3a.2. Find exact values for all remaining trig ratios given that $\sec A = \frac{4}{3}$.

Answer

## 3b. 45°–45°–90° and 30°–60°–90° Triangles Review

The two types of triangles mentioned in the title often appear in calculus. For the acute angles in these triangles, we will find the exact values of the associated trigonometric functions by studying the relationships between the functions, the lengths of the sides, and the angles. We begin with Fig. 3b.1.

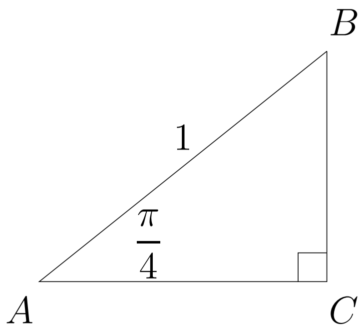



Fig. 3b.1

Given:

$$AB = 1$$
$$\measuredangle A = \frac{\pi}{4} \text{ rad } = 45°$$
$$\measuredangle C = \frac{\pi}{2} \text{ rad } = 90°.$$

Since the sum of the interior angles is $\pi$ or 180°, $\measuredangle B$ also $= \dfrac{\pi}{4}$ radians. This means that the triangle is isosceles, which implies $AC = BC$. By the Pythagorean Theorem,

$$(AC)^2 + (BC)^2 = 1,$$
$$2(BC)^2 = 1,$$
$$BC = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = AC.$$

Thus we have

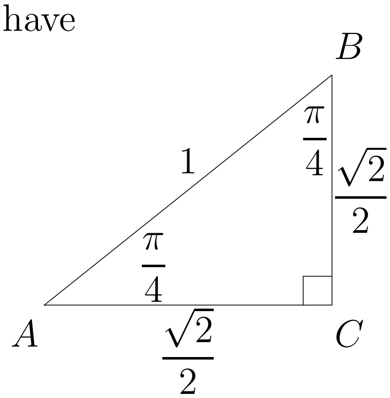



Fig. 3b.2

This implies that $\sin\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$,
$\cos\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$,
$\tan\dfrac{\pi}{4} = 1 = \cot\dfrac{\pi}{4}$, etc.

Next consider a 30° - 60° - 90° triangle.

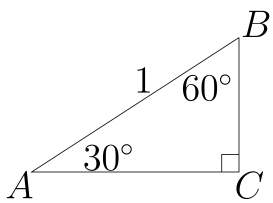



Fig. 3b.3

Given:

$$\measuredangle A = 30° = \frac{\pi}{6} \text{ rad}$$
$$\measuredangle B = 60° = \frac{\pi}{3} \text{ rad}$$
$$\measuredangle C = 90° = \frac{\pi}{2} \text{ rad}$$
$$AB = 1$$

We now form Fig. 3b.4, which comes from flipping the triangle in Fig. 3b.3 about the line $AC$.

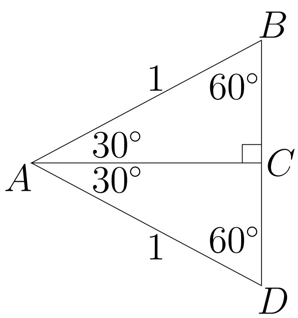



Fig. 3b.4

Evidently, $\triangle ABD$ is an equilateral triangle since each vertex angle is 60°. This means $BD = 1$ and $BC = CD = \dfrac{1}{2}$, since $AC$ is perpendicular to $BD$. (*See remark below.)

So we have

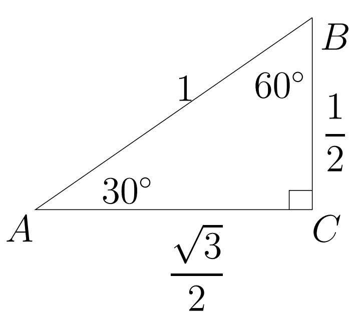



Fig. 3b.5

The Pythagorean Theorem enables us to write $AC = \dfrac{\sqrt{3}}{2}$. Specifically, $1^2 = \left(\dfrac{1}{2}\right)^2 + (AC)^2$, or $AC = \dfrac{\sqrt{3}}{2}$. Using right triangle definitions (Review Topic 3a), we see that $\sin 30° = \sin \dfrac{\pi}{6} = \dfrac{1}{2}$, $\cos 30° = \cos \dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$, $\sin 60° = \sin \dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$, $\cos 60° = \cos \dfrac{\pi}{3} = \dfrac{1}{2}$, $\tan 30° = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$, $\tan 60° = \dfrac{\sqrt{3}}{1} = \sqrt{3}$, etc.

***Remark:** There is a popular formula that says the side opposite the 30° angle in a right triangle is one half the hypotenuse. Now we see why this is correct.

Based on Figures 3b.2 and 3b.5, we can fill in the following table.

| $\theta$ | $\dfrac{\pi}{6}$ | $\dfrac{\pi}{4}$ | $\dfrac{\pi}{3}$ |
|---|---|---|---|
| $\sin\theta$ | $\dfrac{1}{2}$ | $\dfrac{1}{\sqrt{2}}$ | $\dfrac{\sqrt{3}}{2}$ |
| $\cos\theta$ | $\dfrac{\sqrt{3}}{2}$ | $\dfrac{1}{\sqrt{2}}$ | $\dfrac{1}{2}$ |
| $\tan\theta$ | $\dfrac{1}{\sqrt{3}}$ | $1$ | $\sqrt{3}$ |

Table 3b.6

PRACTICE PROBLEMS for Topic 3b-45°–45°–90° and 30°–60°–90° Triangles

3b.1. Study Figs. 3b.2 and 3b.5 and be able to reconstruct them by memory. Try to understand how the various quantities are derived.

3b.2 From memory, fill in Table 3b.6. Answer

3b.3 Construct an isosceles right triangle with legs of length 4. Find the sine, cosine, and tangent of $\frac{\pi}{4}$ and compare these values to those found in Fig. 3b.2. What conclusions can be drawn? Answer

## 3c. Determining Lengths of Sides

Right triangle trigonometry can also give the lengths of the sides of a right triangle, if you know at least one of the acute angles and one side of the triangle. For example, in the figure below, find $BC$ and $AC$.

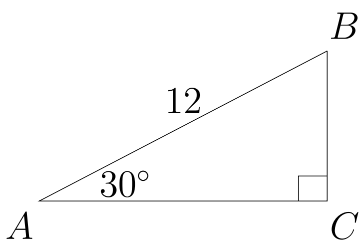



Fig. 3c.1

Since $\sin 30^\circ = \sin\frac{\pi}{6} = \frac{1}{2}$ (see Review Topic 3b), we can write $\frac{1}{2} = \sin A = \frac{BC}{12}$, or $BC = 6$. Then $AC$ can be obtained by suing the cosine or the tangent function (or the Pythagorean Theorem). For example, $\frac{1}{\sqrt{3}} = \tan A = \frac{BC}{AC}$. Thus $\frac{1}{\sqrt{3}} = \frac{6}{AC}$, or $AC = 6\sqrt{3}$.

Note that since $A = 30^\circ$ (and $C = 90^\circ$), angle $B = 60^\circ$. Then we could apply the same reasoning to angle $B$ and obtain the lengths of the respective sides.

Let's do another example where you have to use a calculator. Consider

Fig. 3c.2, where angle $B = 50^\circ$ and $AC = 4$. Find the lengths of $AB$ and $BC$.

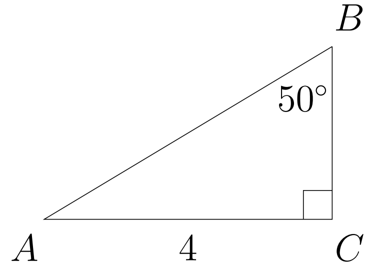



Fig. 3c.2

Using a calculator (set in degree mode!) $\sin 50^\circ = .766$. Thus,

$$.766 = \sin 50^\circ = \sin B = \frac{4}{AB}, \text{ or}$$
$$AB = \frac{4}{.766} = 5.22 \text{ (approximately).}$$

Next,

$$\tan B = \tan 50^\circ = 1.19 = \frac{4}{BC}, \text{ or}$$
$$BC = \frac{4}{1.19} = 3.36.$$

In calculus, sometimes the following problem is encountered. If $\sin\theta = x$, and $\theta$ is in the first quadrant, find $\cos\theta$ and $\tan\theta$. To solve this problem, we first note that $x = \frac{x}{1}$. This allows us to create the following picture, since $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$.

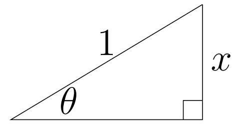



Fig. 3c.3

By the Pythagorean Theorem, we can solve for the missing side (call it $a$), at least algebraically. That is, $c^2 = a^2 + b^2$, or $1 = a^2 + x^2$, or $a = \sqrt{1 - x^2}$.

So we now have the following.

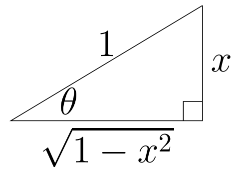



Fig. 3c.4

This means $\cos\theta = \sqrt{1-x^2}$, and $\tan\theta = \dfrac{x}{\sqrt{1-x^2}}$.

PRACTICE PROBLEMS for Topic 3c-Determining Lengths of Sides

3c.1.

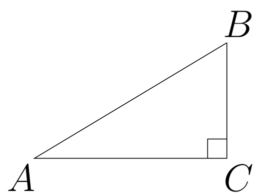



Fig. 3c.5

Suppose $A = 45° = \dfrac{\pi}{4}$ radians. If $BC = 4$, find $AC$ and $AB$. Answer

3c.2.

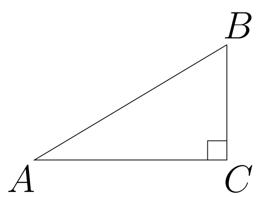



Fig. 3c.6

Suppose $AB = 8$ and angle $A = 1$ radian. Find $AC$ and $BC$. Answer

3c.3. If $\cos\theta = \dfrac{x}{2}$ and $\theta$ is in the first quadrant, find $\sin\theta$ and $\tan\theta$. Answer

## 3d. Reference Angles and the Trigonometric Functions

The definitions of the trigonometric functions in Review Topic 3a were based on having an acute angle (less than $90^\circ$). What happens if $A > 90^\circ = \dfrac{\pi}{2}$ radians? For example, Let $A$ be the angle pictured in Fig. 3d.1. Now, what is meant by $\sin A$, $\cos A$, etc.?

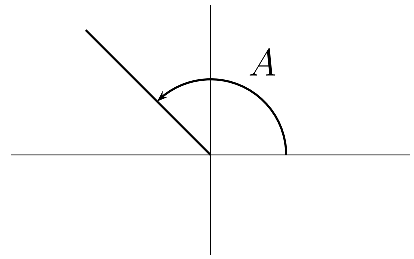



Fig. 3d.1

To answer this question, we consider the reference angle of $A$, which is the acute angle between the terminal side of $A$ and the $x-$axis. In this case, the reference angle is $(\pi - A)$ as in Fig. 3d.2. (For the rest of our discussion, the reference angle of $A$ will also be denoted by ref A.)

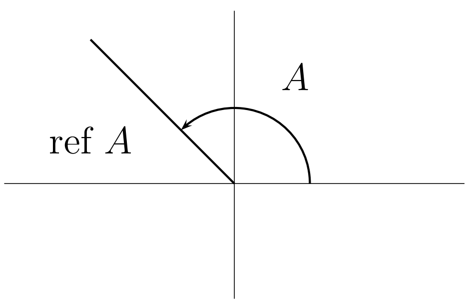



Fig. 3d.2

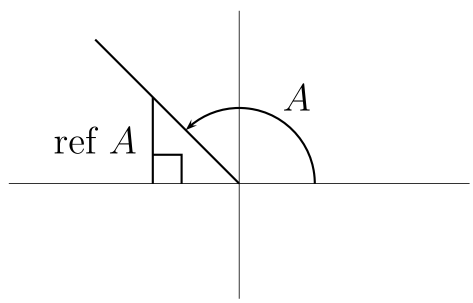



Fig. 3d.3

We next form a right triangle between the terminal side of $A$ and the $x-$axis, as indicated in Fig. 3d.3. Then we define

$$\sin A = \sin(\text{ref angle}).$$

Note that since (ref $A$) is an acute angle with its own right triangle, we can again use definitions involving an opposite side, adjacent side, etc., just as we did for Fig. 3a.1 in Review Topic 3a.

For example, consider Fig. 3d.4 below.

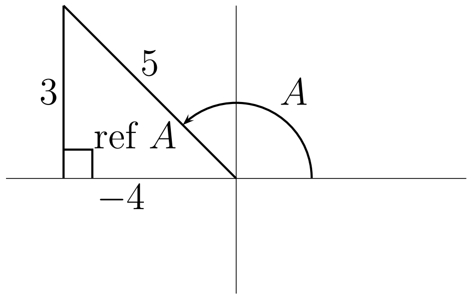



Fig. 3d.4

Then we define $\sin A = \sin(\text{reference angle}) = \frac{3}{5}$, $\cos A = -\frac{4}{5}$, $\tan A = \frac{3}{-4} = \frac{-3}{4}$, etc. Observe that the adjacent side of the reference angle is $(-4)$, since $x$ is negative. Also note that since we have a right triangle, the numbers 3, $-4$, and 5 satisfy the Pythagorean Theorem.

The situation is similar if the terminal side of $A$ lies in the third or fourth quadrant, as in Fig. 3d.5.

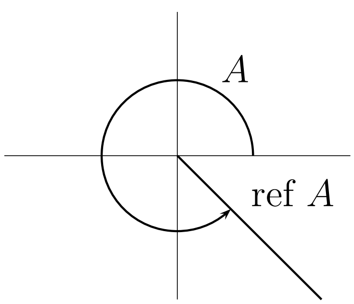



Fig. 3d.5

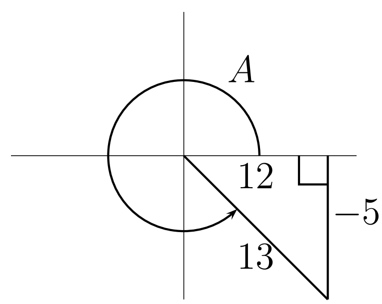



Fig. 3d.6

We form a right triangle with the reference angle as indicated in Fig. 3d.6, and we suppose the lengths of the sides are as shown. Then
$\sin A = \sin(\text{reference angle}) = \frac{-5}{13}$, $\cos A = \frac{12}{13}$, $\tan A = \frac{-5}{13}$, etc.

PRACTICE PROBLEMS for Topic 3d-Reference Angles and the Trigonometric Functions

3d.1. Consider angle $A$ as in Fig. 3d.7, and suppose its reference angle forms a right triangle with the dimensions shown in Fig. 3d.8. Find $\sin A,\ \cos A,\ \tan A$.

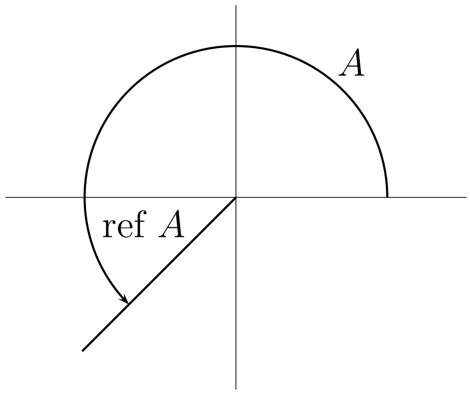



Fig. 3d.7

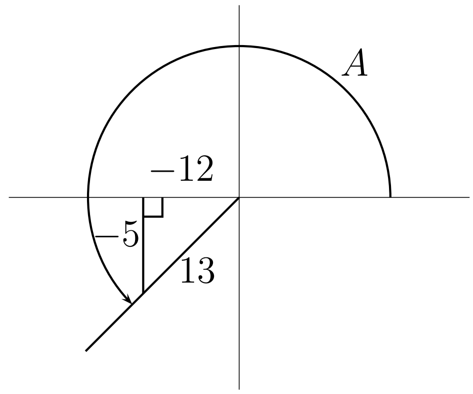



Fig. 3d.8

Answer

Evidently, based on Figures 3.d4, 3d.6, and 3d.8, we can conclude that $\sin\theta > 0$ for $\theta$ in the first and second quadrants and $\sin\theta < 0$ for $\theta$ in the third and fourth quadrants. Similarly, $\cos\theta$ is positive in the first and fourth quadrants and negative in the second and third quadrants, while $\tan\theta$ is positive in the first and third quadrants and negative in the second and fourth quadrants. The table below summarizes these observations.

| | 1st Q. | 2nd Q. | 3rd Q. | 4th Q. |
|---|---|---|---|---|
| $\sin\theta$ | + | + | − | − |
| $\cos\theta$ | + | − | − | + |
| $\tan\theta$ | + | − | + | − |

Table 3d.9 -“Q” means quadrant.

PRACTICE PROBLEMS for Topic 3d-Continued

3d.2. Determine whether the following expressions are positive or negative.

a) $\tan \dfrac{7\pi}{6}$, b) $\sec \dfrac{3\pi}{4}$, c) $\sin \dfrac{11\pi}{6}$, d) $\sin \dfrac{5\pi}{6}$,

e) $\cos \dfrac{5\pi}{4}$, f) $\tan \theta$, if $\dfrac{3\pi}{2} < \theta < 2\pi$.

Answers

3d.3. Name the quadrant(s) where $\theta$ must terminate, given the following information.

a) $\sin \theta < 0$

b) $\sin \theta > 0$ and $\tan \theta < 0$.

Answers

Finally, we can use the idea of reference angles to extend what we know about 45° - 45° - 90° and 30° - 60° - 90° triangles from the first quadrant to the other quadrants. For example, let $A = \dfrac{\pi}{6}$. Let $B$, $C$, and $D$ be the positive angles in the second, third, and fourth quadrants respectively that have reference angle $\dfrac{\pi}{6}$. A moment's reflection yields that $B = \dfrac{5\pi}{6}$, $C = \dfrac{7\pi}{6}$, and $D = \dfrac{11\pi}{6}$.

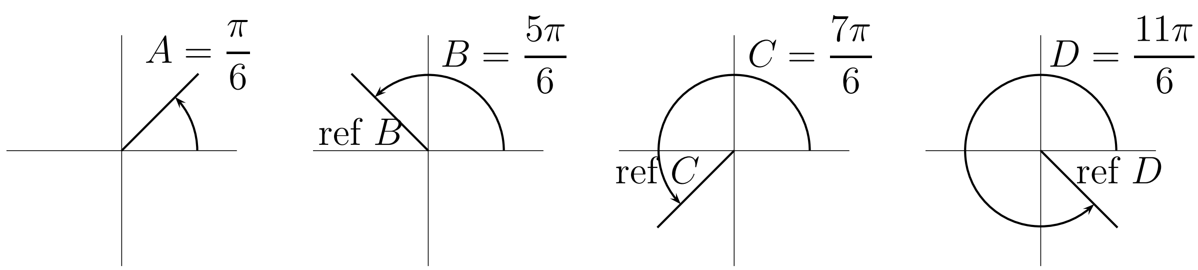



Fig. 3d.10. All reference angles above $= \dfrac{\pi}{6}$.

As shown in Review Topic 3b, $\sin\frac{\pi}{6} = \frac{1}{2}$. This leads to Fig. 3d.11.

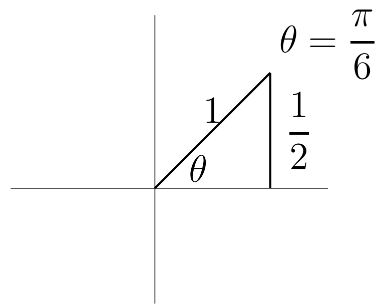



Fig. 3d.11

Figs. 3d.10 and 3d.11 immediately yield the following information.

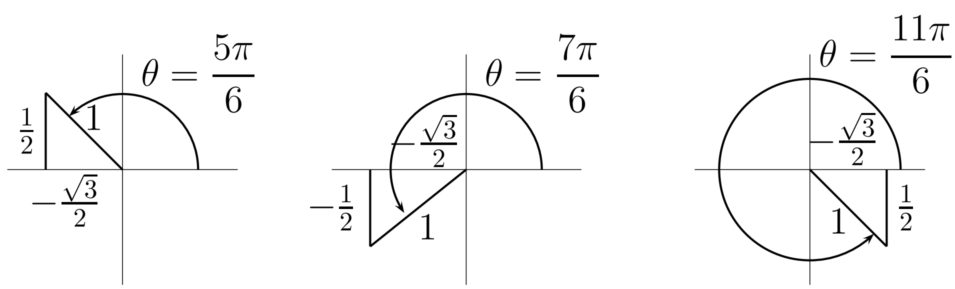



Fig. 3d.12.

Fig. 3d.12 enables us to create the table below.

| $\theta$ | $\frac{5\pi}{6}$ | $\frac{7\pi}{6}$ | $\frac{11\pi}{6}$ |
|---|---|---|---|
| $\sin\theta$ | $\frac{1}{2}$ | $\frac{-1}{2}$ | $-\frac{1}{2}$ |
| $\cos\theta$ | $-\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{2}$ |
| $\tan\theta$ | $-\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $-\frac{1}{\sqrt{3}}$ |

Table 3d.13

We remark that similar reasoning applies for negative angles. For example, if $\theta = -\dfrac{\pi}{6}$, it has the same reference angle as the drawing on the right in Fig. 3d.12 $\left(\text{when } \theta = \dfrac{11\pi}{6}\right)$. Thus, $\sin\left(-\dfrac{\pi}{6}\right) = -\dfrac{1}{2}$, $\cos\left(-\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$, etc. As an additional example, $\theta = -\dfrac{3\pi}{4}$ would have the same reference angle as $\theta = \dfrac{5\pi}{4}$.

Another observation is that $\sin\left(-\dfrac{\pi}{6}\right) = -\sin\left(\dfrac{\pi}{6}\right)$ and $\cos\left(-\dfrac{\pi}{6}\right) = \cos\left(\dfrac{\pi}{6}\right)$. We will see in Review Topic 13 that this observation is true in general. That is, for any $x$, $\sin(-x) = -\sin x$ ($\sin x$ is an odd function), $\cos(-x) = \cos x$ ($\cos x$ is an even function), and $\tan(-x) = -\tan x$ ($\tan x$ is an odd function). The concept of even and odd functions is explained in Review Topic 1.

PRACTICE PROBLEMS for Topic 3d-Continued

3d.4. What are the positive angles in the second, third, and fourth quadrants that have $\dfrac{\pi}{3}$ as their reference angle? Answer

3d.5. What are the positive angles in the second, third, and fourth quadrants that have $\dfrac{\pi}{4}$ as their reference angle? Answer

3d.6. Arguing as we did in Fig. 3d.10, 3d.11, and 3d.12, complete the tables below.

| $\theta$ | $\dfrac{2\pi}{3}$ | $\dfrac{4\pi}{3}$ | $\dfrac{5\pi}{3}$ |
|---|---|---|---|
| $\sin\theta$ | | | |
| $\cos\theta$ | | | |
| $\tan\theta$ | | | |

Table 3d.14

3d.6. Continued

| $\theta$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{4}$ | $\frac{7\pi}{4}$ |
|---|---|---|---|
| $\sin\theta$ | | | |
| $\cos\theta$ | | | |
| $\tan\theta$ | | | |

Table 3d.15

Answers

3d.7. Find the exact value for each expression.

a) $\cos\left(-\frac{\pi}{3}\right)$ b) $\tan\left(-\frac{\pi}{4}\right)$ c) $\sin\left(-\frac{3\pi}{4}\right)$ Answers

Beginning of Topic 109 Study Topics 109 Skills Assessment

ANSWERS to PRACTICE PROBLEMS (Topic 3a-Right Triangle Definitions of the Trigonometric Functions)

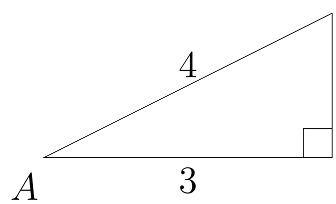
3a.1. a) $\sin A = \frac{3}{5}$; b) $\cos A = \frac{4}{5}$; c) $\tan A = \frac{3}{4}$; d) $\sin B = \frac{4}{5}$;

e) $\cos B = \frac{3}{5}$; f) $\cot B = \frac{3}{4}$; g) $\sec A = \frac{5}{4}$; h) $\sin^2 A + \cos^2 A = 1$;

i) $\csc B = \frac{5}{4}$; j) $\sin\left(\frac{\pi}{2} - A\right) = \frac{4}{5}$;

Return to Problem

3a.2. $\sec A = \frac{4}{3}$ implies we have the following picture.




The Pythagorean Theorem implies that the missing side must equal $\sqrt{16-9} = \sqrt{7}$. Thus, $\sin A = \frac{\sqrt{7}}{4}$, $\cos A = \frac{3}{4}$, $\tan A = \frac{\sqrt{7}}{3}$, $\cot A = \frac{3}{\sqrt{7}}$, and $\csc A = \frac{4}{\sqrt{7}}$.

Return to Problem

ANSWERS to PRACTICE PROBLEMS (Topic 3b–45°–45°–90° and 30°–60°–90° Triangles – Answers)

3b.2

| $\theta$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ |
|---|---|---|---|
| $\sin\theta$ | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ |
| $\cos\theta$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ |
| $\tan\theta$ | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ |

Answer 3b.2

Return to Problem

3b.3 The values of the trigonometric functions in Fig. 3b.2 will be the same for any isoceles right triangle.

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ANSWERS to PRACTICE PROBLEMS (Topic 3c-Determining Lengths of Sides)

3c.1. $\tan\frac{\pi}{4} = 1$. Also, $\tan\frac{\pi}{4} = \frac{BC}{AC}$. Thus, $\frac{BC}{AC} = 1 \Rightarrow AC = 4$. Note that since the triangle is isosceles, elementary geometry also implies that $AC = 4$. Next $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{BC}{AB} = \frac{4}{AB}$. Thus $AB = 4\sqrt{2}$.

(**Reminder:** We can also find $AB$ by using the Pythagorean Theorem.)

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3c.2. Using a calculator set in radian mode, we find that $\sin(1) = .84$. This means $\frac{BC}{8} = .84$, or $BC = 6.73$. Next, $\cos(1) = .54$. Since $\cos 1 = \frac{AC}{AB} = \frac{AC}{8}$, we have $AC = 8(.54) = 4.32$.

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3c.3. Given $\cos\theta = \frac{x}{2}$ we can draw the following picture.

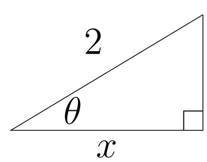


Fig. 3c.7

The Pythagorean Theorem implies the missing side has length $\sqrt{4-x^2}$.

Then,

$$\sin\theta = \frac{\sqrt{4-x^2}}{2}, \quad \text{and} \quad \tan\theta = \frac{\sqrt{4-x^2}}{x}.$$

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ANSWERS to PRACTICE PROBLEMS (Topic 3d-Reference Angles and the Trigonometric Functions)

3d.1. $\sin A = -\dfrac{5}{13}, \cos A = -\dfrac{12}{13}, \tan A = \dfrac{-5}{-12} = \dfrac{5}{12}.$

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3d.2. a) positive b) negative, since $\sec\theta = \dfrac{1}{\cos\theta.}$

c) negative d) positive e) negative f) negative

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3d.3. a) 3rd quadrant or 4th quadrant

b) 2nd quadrant

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3d.4. $\dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

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3d.5. $\dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

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ANSWERS to PRACTICE PROBLEMS (Topic 3d-Reference Angles and the Trigonometric Functions)

3d.6.

| $\theta$ | $\frac{2\pi}{3}$ | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ |
|---|---|---|---|
| $\sin\theta$ | $\frac{\sqrt{3}}{2}$ | $\frac{-\sqrt{3}}{2}$ | $\frac{-\sqrt{3}}{2}$ |
| $\cos\theta$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ |
| $\tan\theta$ | $-\sqrt{3}$ | $\sqrt{3}$ | $-\sqrt{3}$ |

Table 3d.15

| $\theta$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{4}$ | $\frac{7\pi}{4}$ |
|---|---|---|---|
| $\sin\theta$ | $\frac{\sqrt{2}}{2}$ | $\frac{-\sqrt{2}}{2}$ | $\frac{-\sqrt{2}}{2}$ |
| $\cos\theta$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ |
| $\tan\theta$ | $-1$ | $1$ | $-1$ |

Table 3d.15

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ANSWERS to PRACTICE PROBLEMS (Topic 3d-Reference Angles and the Trigonometric Functions)

3d.7. a) $\dfrac{1}{2}$, b) $-1$ c) $-\dfrac{\sqrt{2}}{2}$.

NOTE: The solution for part a) can be derived in two ways.

First, if we use the concept of reference angle, then $\theta = -\dfrac{\pi}{3}$ has reference angle $\dfrac{\pi}{3}$. From Table 3b.6, $\cos\dfrac{\pi}{3} = \dfrac{1}{2}$. Since $-\dfrac{\pi}{3}$ is in the fourth quadrant where the $x$ coordinate is positive, we have $\cos\left(-\dfrac{\pi}{3}\right) = \dfrac{1}{2}$.

As a second approach, since $\cos x$ is an even function, (i.e., $\cos(-x) = \cos x$, we can simply say $\cos\left(-\dfrac{\pi}{3}\right) = \cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$. Similar comments apply for answers b) and c).

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