MATH 150 – TOPIC 13

GRAPHS OF SIN X, COS X, AND TAN X

In Review Topic 9a, the trigonometric "functions" were introduced as ratios, while in Review Topic 11 they were derived by considering points on the unit circle. In this section we focus on their behavior as functions and study their graphs.

The graphs of $\sin x$, $\cos x$, and $\tan x$ are given below, where x must be in radians. The table in Review Topic 12 is a summary of exact values that can be used to obtain graphs.
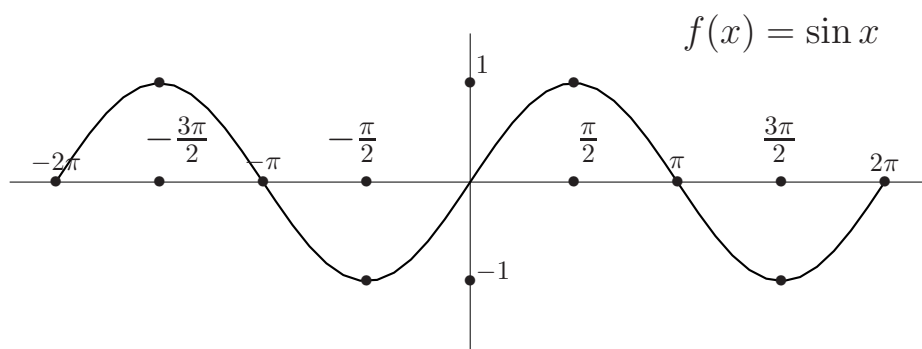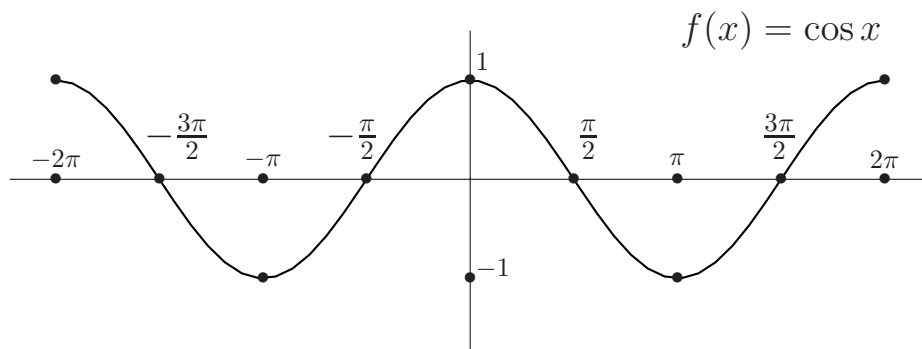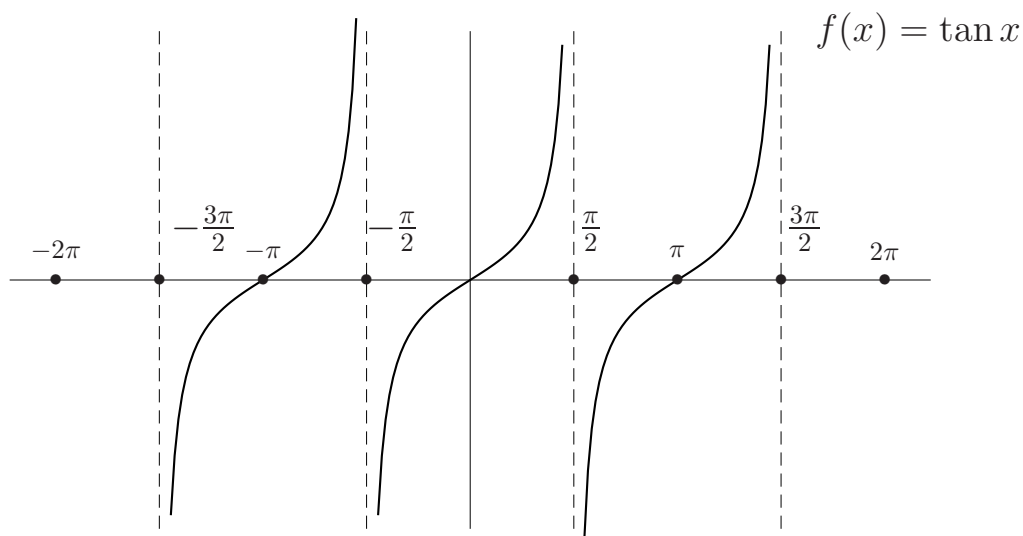

Fig. 13.1

Fig. 13.2

$$f(x) = \tan x$$

Fig. 13.3

The graphs enable us to make several conclusions. First, $\sin x$ and $\cos x$ are periodic with period 2π, while $\tan x$ is periodic with period π. (See also Review Topic 11.)

The concept of even and odd functions (Review Topic 1) applies here. Evidently, $\sin x$ and $\tan x$ are odd functions since they are symmetric with respect to the origin. Thus

$$\sin(-x) = -\sin x, \text{ and } \tan(-x) = -\tan x.$$

Example: $\sin\left(-\dfrac{5\pi}{6}\right) = -\sin\left(\dfrac{5\pi}{6}\right) = -\dfrac{1}{2}$, and
$\tan\left(-\dfrac{3\pi}{4}\right) = -\tan\left(\dfrac{3\pi}{4}\right) = -(-1) = 1.$ (Review Topic 12)

The graph of $\cos x$ implies it is an even function since it is symmetric with respect to the y axis. This means

$$\cos(-x) = \cos x.$$

Example: $\cos\left(-\dfrac{2\pi}{3}\right) = \cos\left(\dfrac{2\pi}{3}\right) = -\dfrac{1}{2}$. (Review Topic 12)

The table below summarizes the previous discussion.

	Period	Symmetry
$\sin x$	2π	odd
$\cos x$	2π	even
$\tan x$	π	odd

Table 13.4

PRACTICE PROBLEM for Topic 13 – Graphs of $\sin x$, $\cos x$, and $\tan x$

13.1   On a blank sheet of paper, by memory, draw the graphs of $\sin x$, $\cos x$, and $\tan x$ on the interval $-2\pi \le x \le 2\pi$. Label all x and y intercepts, maxima and minima (high and low points), and vertical asymptotes.

Answer. See Figures 13.1, 13.2, and 13.3.

Beginning of Topic   Skills Assessment