MATH 250 – REVIEW TOPIC 3

Partial Fraction Decomposition and Irreducible Quadratics

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Certain integration techniques in Calc II require the use of an algebraic process called partial fraction decomposition. This review topic introduces you to the algebra required in this process.

I. Decompositions with Linear Factors

A. Partial fraction decomposition is a technique used to transform algebraic expressions into equivalent forms. Our discussion begins with finding the sum of $\dfrac{3}{x+2}$ and $\dfrac{2}{x-1}$.

$$\text{Ans: } \frac{3}{x+2} + \frac{2}{x-1} = \frac{3(x-1)+2(x+2)}{(x+2)(x-1)} = \frac{5x+1}{(x+2)(x-1)}.$$

Question: Is the summing of fractions reversible? In other words, can we decompose or “break up” the fraction? Specifically, can we find fractions whose sum is $\dfrac{5x+1}{(x+2)(x-1)}$?

Yes, this is possible. Let us study several examples and determine the process.

Example: Decompose $\dfrac{5x+1}{(x+2)(x-1)}$.

Based on the factors appearing in the denominator, we would expect our decomposition to have denominators using these same factors (basic LCD principles). By selecting A and B to represent the unknown numerators, the decomposition can be stated as follows:

$$\frac{5x+1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}.$$

What remains is to find values for A and B.

Start by clearing fractions (multiplying each side by $(x+2)(x-1)$). We get

$$5x + 1 = A(x-1) + B(x+2),$$

an equation which is true for all x.

What if we select $x = 1$. This yields $6 = A(0) + B(3) \Rightarrow B = 2$.

What other x value would make a good selection?

How about $x = -2$. This gives $-9 = A(-3) + B(0) \Rightarrow A = 3$.

Note: Selecting values of x that eliminate factors is the key. What would happen if other selections were made? We suggest you investigate.

Result: We have successfully decomposed $\dfrac{5x+1}{(x+2)(x-1)}$.

$$\begin{aligned}\frac{5x+1}{(x+2)(x-1)} &= \frac{A}{x+2} + \frac{B}{x-1} \text{ with } A = 3,\ B = 2\\ &= \frac{3}{x+2} + \frac{2}{x-1}.\end{aligned}$$

Exercise 1: Decompose a) $\dfrac{x+4}{x(x-2)}$ b) $\dfrac{x+2}{x^2+4x+3}$. Answers

B. Sometimes a denominator contains a repeated factor.

Illustration:

First, consider $\dfrac{1}{x(x-1)(x+1)}$, which has no repeating factors. This decomposes into $\dfrac{A}{x}+\dfrac{B}{x-1}+\dfrac{C}{x+1}$. Each factor is distinct and contributes a term in the decomposition.

Now consider $\dfrac{1}{x(x-1)^2}$, a fraction whose denominator contains a repeated factor. The decomposition is

$$\frac{1}{x(x-1)^2}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{(x-1)^2}.$$

In a similar fashion

$$\frac{2x+1}{(x+1)^2(x-1)^3}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-1}+\frac{D}{(x-1)^2}+\frac{E}{(x-1)^3}.$$

Exercise 2: Decompose $\dfrac{5x^2+20x+6}{x^3+2x^2+x}$. Answer

Hint: Start by factoring the denominator.

C. It's time we show an alternate method for finding constants. Consider the equation $4x^2+7x-3=Ax^2+Bx+C$.

Such an equation is true for all values of x only when corresponding terms of both polynomials are equivalent.

$$4x^2+7x-3=Ax^2+Bx+C \Rightarrow \begin{array}{l} A=4 \\ B=7 \\ C=-3 \end{array}$$

Exercise 3: Find values for the constants A, B , and C.

a) $3x^2 - 4 = Ax^2 + Bx + C$

b) $1 = Ax^2 + (B + C)x + C$

Answers

We'll refer to this as Comparing Coefficients. Let's redo our first example using this method.

Example. Decompose $\dfrac{5x+1}{(x+2)(x-1)}$.

$$\frac{5x+1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$
$$\Rightarrow 5x + 1 = A(x-1)+B(x+2)$$
$$\left.\begin{aligned} &\Rightarrow 5x + 1 = Ax + Bx - A + 2B \\ &\Rightarrow 5x + 1 = (A+B)x+(-A+2B) \end{aligned}\right\} \text{Collecting like terms}$$

Now compare coefficients:

$$5x + 1 = (\underbrace{A + B})x + (\overbrace{-A + 2B})$$

$$A + B = 5 \quad \text{and} \quad -A + 2B = 1$$

Solving the system gives the same result as before, namely $A = 3$ and $B = 2$.

Exercise 4: Decompose each expression.

a) $\dfrac{2x^2 + x - 12}{x(x+3)(x+2)}$ (b) $\dfrac{4x^2 + 2x - 1}{x^3 + x^2}$

Find constants by comparing coefficients.

Answer (a) Answer (b)

Practice Problems.

Decompose the following:

3.1. $\dfrac{3}{x^2+x-2}$ Answer

3.2. $\dfrac{5-x}{2x^2+x-1}$ Answer

3.3. $\dfrac{3x+4}{(x+2)^2}$ Answer

3.4. $\dfrac{12}{x^4-x^3-2x^2}$ Answer

II. A. Irreducible Quadratics

Our discussion has been limited to linear factors. There is another type of factor to consider called an irreducible quadratic.

Question: What is an Irreducible Quadratic?

Clearly x^2+4x+3 is quadratic and reduces (factors).

We say a quadratic ($\mathbf{ax^2+bx+c}$) is irreducible if it cannot be factored over the real numbers.

What about x^2-5 or x^2+4 or x^2+4x-3. Which of these is irreducible?

Only x^2+4. Both x^2-5 and x^2+4x-3 can be factored over the reals.

$$x^2-5=(x-\sqrt{5})(x+\sqrt{5})$$

$$x^2+4x-3=(x+2+\sqrt{7})(x+2-\sqrt{7})$$

Check:

$$\begin{aligned}(x+2+\sqrt{7})(x+2-\sqrt{7}) &= (x+2)^2-7 \\ &= x^2+4x-3\end{aligned}$$

This isn't typical factoring, but it is acceptable given the condition "over the reals".

Question: How can we show $x^2 + 4$, or any quadratic, is irreducible?

A quadratic with real roots has real factors and thus is reducible. Only a quadratic with complex roots, like $x^2 + 4$, is irreducible. The nature of roots is best determined by the discriminant, $b^2 - 4ac$, from the quadratic formula.

Given a quadratic $ax^2 + bx + c$:

$$b^2 - 4ac < 0 \Rightarrow \text{complex roots} \Rightarrow \text{irreducible quadratic}$$

$$b^2 - 4ac \geq 0 \Rightarrow \text{real roots} \Rightarrow \text{reducible quadratic}$$

Exercise 5: Which of the following is irreducible?

a) $x^2 - 10$ b) $x^2 + 9$ c) $x^2 + 2x + 4$ Answers

B. Decompositions with Irreducible Quadratics

Let's now set up a decomposition using an irreducible quadratic.

Example. What is the correct form of the decomposition for

$$\frac{1}{x(x^2+1)},$$

given that x is linear and $(x^2 + 1)$ is an irreducible quadratic?

Answer: $\dfrac{1}{x(x^2+1)} = \dfrac{A}{x} + \dfrac{Bx + C}{x^2 + 1}$

Note: linear factors have constants in the numerator, while irreducible quadratics have numerators of the form $(\quad)x + (\quad)$.

Solving for constants is no different than before: clear fractions, then select x's or compare coefficients.

Here is the entire decomposition worked out.

$$\begin{aligned}\frac{1}{x(x^2+1)} &= \frac{A}{x} + \frac{Bx+C}{x^2+1}\\ &\Rightarrow 1 = A(x^2+1) + (Bx+C)x\\ &\Rightarrow 1 = x^2(A+B) + Cx + A.\end{aligned}$$

Setting coefficients equal:

$$A+B=0, \quad C=0, \quad A=1 \Rightarrow B=-1.$$

Thus $\dfrac{1}{x(x^2+1)} = \dfrac{1}{x} + \dfrac{-x}{x^2+1}$.

Exercise 6: Set up the decomposition for $\dfrac{x^2+9}{x^4-2x^2-8}$. Answer

If setting up the decomposition is still unclear, compare the following.

Illustration:

$$\frac{x+1}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3} \qquad x^2+3 \text{ is irreducible}$$

$$\frac{3x-2}{x(x^2-3)} = \frac{A}{x} + \frac{B}{x-\sqrt{3}} + \frac{C}{x+\sqrt{3}} \qquad x^2-3 \text{ is reducible}$$

$$\frac{2x^2+x-4}{x^3-3x^2} = \frac{1}{x^2(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3} \qquad x^2 \text{ is linear and repeated}$$

$$\frac{3x^3-2x^2+4x-1}{(x^2+3)^2} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{(x^2+3)^2} \qquad x^2+3 \text{ is irreducible and repeated}$$

Exercise 7: Decompose $\dfrac{2x-1}{x(x^2+4)}$. Answer

Practice Problems.

A. Write the form of the partial fraction decomposition. DO NOT SOLVE for the constants.

3.5. $\dfrac{2x-3}{x^3+10x}$ Answer

3.6. $\dfrac{1}{(x^2-2)^2}$ Answer

B. Decompose the following:

3.7. $\dfrac{2x-3}{x^3+10x}$ Answer

3.8. $\dfrac{x+4}{x^4+3x^2-4}$ Answer

III. Factoring Cubic Polynomials

This section is intended to help you with factoring, in particular factoring cubic polynomials. What if you had to decompose A) $\dfrac{6x^2+x+1}{x^3-x^2+x-1}$ or B) $\dfrac{1}{x^3-2x-4}$ or C) $\dfrac{2x+4}{x^3-1}$? The first step would be to factor the denominator. Let's take these one at a time.

Example A: Grouping would work.

$$x^3-x^2+x-1 = x^2(x-1)+(x-1) = (x^2+1)(x-1).$$

This means to set up the decomposition:

$$\frac{6x^2+x+1}{x^3-x^2+x-1} = \frac{6x^2+x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}\,.$$

Example B: Factoring any polynomial like x^3-2x-4 presents a greater challenge. We suggest the Factor Theorem.

> **Factor Theorem:** If a polynomial $p(x)$ has a root at $x=c$, i.e., $p(c)=0$, then $x-c$ is a factor of $p(x)$.

Here's how to factor x^3-2x-4 using this theorem.

Solution: We must first find a root. Any factor of 4 $(\pm 1, 2, 4)$ is a candidate. Since $p(2) = 0$, 2 is a root and $(x-2)$ is a factor. To get the other factor, simply divide:

$$(x-2)\overline{\big)\, x^3 - 2x - 4} = x^2 + 2x + 2.$$

That means $x^3 - 2x - 4 = (x-2)(x^2+2x+2)$ and

$$\frac{1}{x^3 - 2x - 4} = \frac{1}{(x-2)(\underbrace{x^2+2x+2}_{b^2-4ac=-4})} = \frac{A}{x-2} + \frac{Bx+C}{x^2+2x+2}.$$

Example C: Factor as the difference of cubes.

$$x^3 - 1 = (x-1)(x^2+x+1).$$

Notice that $x^2 + x + 1$ is irreducible: $b^2 - 4ac = 1 - 4 = -3$. If you have problems with cube factoring, the Factor Theorem is still available. Since $x = 1$ is a root, $(x-1)$ is a factor, and the quotient from $(x-1)\overline{\big)\, x^3 - 1}$ is the other factor.

Note: The Factor Theorem would also work also on Example A. Try it out yourself.

Exercise 8: Express as a product of linear and/or irreducible quadratic factors.

a) $x^3 + 3x^2 - 2x - 6$
b) $x^4 - x$
c) $x^3 - x + 6$

Answers

Final Note: There is one other issue we need to address. Only "proper" fractions (degree of numerator is less than degree of denominator) can be decomposed. That means to decompose $\dfrac{x^2+1}{x^2-1}$ we first must divide:
$(x^2-1)\overline{\big)\, x^2 + 1} = 1 + \dfrac{2}{x^2-1}$. Now you decompose $\dfrac{2}{x^2-1}$.

IV. Guidelines for Fraction Decomposition

Set up:

1) Divide if fraction is improper.

2) Factor the denominator; express as a product of linear and/or irreducible quadratic factors.

3) Each nonrepeated factor contributes a term in the decomposition. Numerators are assigned according to the type of factor:

$$\text{Linear Factors} \rightarrow \text{Numerators of } A, B, C, \ldots$$
$$\text{Irred. Quadratic factors} \rightarrow \text{Numerators of } Ax + B,\ Cx + D,\ \ldots$$

4) If a factor repeats n times, as in $(\quad)^n$, the decomposition has terms with denominators of the form $(\quad)^k$, for each value of $k = 1, \ldots, n$.

Solving for Constants:

5) Multiply by LCD to clear fractions.

6) Substitute values of x and/or compare coefficients.

Answers to Practice Problems.

3.1. $3 = A(x-1) + B(x+2)$; $A = -1$, $B = 1$

$$\frac{3}{x^2 + x - 2} = \frac{-1}{x+2} + \frac{1}{x-1}$$

Return to Problem

3.2. $5 - x = A(x+1) + B(2x-1)$; $B = -2$, $A = 3$

$$\frac{5-x}{(2x-1)(x+1)} = \frac{3}{2x-1} + \frac{-2}{x+1}$$

Return to Problem

3.3. $$\frac{3x+4}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$$

$$3x+4 = A(x+2) + B \Rightarrow A = 3,\ B = -2$$

$$\frac{3x+4}{(x+2)^2} = \frac{3}{x+2} + \frac{-2}{(x+2)^2}$$

Return to Problem

3.4. $$\frac{12}{x^4 - x^3 - 2x^2} = \frac{12}{x^2(x-2)(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} + \frac{D}{x+1}$$
$$= \frac{4}{x} + \frac{-6}{x^2} + \frac{1}{x-2} + \frac{-4}{x+1}$$

Return to Problem

3.5. $$\frac{2x-3}{x^3+10x} = \frac{2x-3}{x(x^2+10)} = \frac{A}{x} + \frac{Bx+C}{x^2+10}$$

Return to Problem

3.6. $$\frac{1}{(x^2-2)^2} = \frac{1}{(x-\sqrt{2})^2(x+\sqrt{2})^2}$$
$$= \frac{A}{x-\sqrt{2}} + \frac{B}{(x-\sqrt{2})^2} + \frac{C}{x+\sqrt{2}} + \frac{D}{(x+\sqrt{2})^2}$$

$x^2 - 2$ reduces into $(x-\sqrt{2})(x+\sqrt{2})$ Return to Problem

3.7 $$\frac{2x-3}{x(x^2+10)} = \frac{A}{x} + \frac{Bx+C}{x^2+10}$$

$$\Rightarrow 2x-3 = A(x^2+10) + (Bx+C)x$$
$$\Rightarrow 2x-3 = (Ax^2+Bx^2) + Cx + 10A$$
$$\Rightarrow 2x-3 = (A+B)x^2 + Cx + 10A \Rightarrow C = 2,\ A = \frac{-3}{10},\ B = \frac{3}{10}$$

$$\frac{2x-3}{x(x^2+10)} = \frac{\frac{-3}{10}}{x} + \frac{\frac{3}{10}x+2}{x^2+10}$$

Return to Problem

3.8. $$\frac{x+4}{x^4+3x^2-4} = \frac{x+4}{(x-1)(x+1)(x^2+4)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+4}$$

$$\Rightarrow x+4 = A(x+1)(x^2+4) + B(x-1)(x^2+4) + (Cx+D)(x^2-1)$$

$$\begin{aligned} &\text{if } x=1, && 5 = 10A \Rightarrow A = \frac{1}{2} \\ &\text{if } x=-1, && 3 = -10B \Rightarrow B = \frac{-3}{10} \end{aligned}$$

We'll find C and D by comparing coefficients

$$\begin{aligned} x+4 = (A+B+C)x^3 + (A-B+D)x^2 &+ (4A+4B-C)x \\ &+ (4A-4B-D) \end{aligned}$$

$$A+B+C = 0 \text{ with } A = \frac{1}{2} \text{ and } B = \frac{-3}{10} \Rightarrow C = \frac{-1}{5}$$

$$A-B+D = 0 \Rightarrow D = \frac{-4}{5}$$

$$\frac{x+4}{x^4+3x^2-4} = \frac{\frac{1}{2}}{x-1} + \frac{\frac{-3}{10}}{x+1} + \frac{\frac{-1}{5}x - \frac{4}{5}}{x^2+4}$$

Return to Problem

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Decompose a) $\dfrac{x+4}{x(x-2)}$ b) $\dfrac{x+2}{x^2+4x+3}$

Answers:

a) $$\frac{x+4}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2} \Rightarrow$$
$$x+4 = A(x-2) + Bx$$
$$\text{if } x = 2,\ 6 = 2B \Rightarrow B = 3$$
$$\text{if } x = 0,\ A = -2$$
$$\frac{x+4}{x(x-2)} = \frac{-2}{x} + \frac{3}{x-2}$$

b) $$\frac{x+2}{x^2+4x+3} = \frac{x+2}{(x+3)(x+1)} = \frac{A}{x+3} + \frac{B}{x+1} \Rightarrow$$

Important: A denominator must be expressed in factored form before you can begin decomposing.

$$x+2 = A(x+1) + B(x+3)$$
$$\text{if } x = -1, \quad B = \frac{1}{2}$$
$$\text{if } x = -3, \quad A = \frac{1}{2}$$
$$\frac{x+2}{x^2+4x+3} = \frac{\frac{1}{2}}{x+3} + \frac{\frac{1}{2}}{x+1}$$

Return to Review Topic

Decompose $\dfrac{5x^2+20x+6}{x^3+2x^2+x}$.

Answer: $$\frac{5x^2+20x+6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$
$$5x^2+20x+6 = A(x+1)^2 + Bx(x+1) + Cx$$

$$\begin{aligned} \text{if } x=-1,&\quad -9=-C \Rightarrow C=9 \\ x=0,&\quad 6=A \\ {}^*\,x=1,&\quad 31=4A+2B+C \Rightarrow B=-1 \end{aligned}$$

$$\frac{5x^2+20x+6}{x(x+1)^2} = \frac{6}{x} + \frac{-1}{x+1} + \frac{9}{(x+1)^2}$$

*With no other good choices for x, choosing any x would allow us to find B (only becase we've determined values for A and C).

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Find the values for all constants:

a) $3x^2 - 4 = Ax^2 + Bx + C$

b) $1 = Ax^2 + (B + C)x + C$

Answers:

a) $A = 3,\ B = 0,\ C = -4$

b) $A = 0,\ C = 1,\ B + C = 0 \Rightarrow B = -1$

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a) Decompose $\dfrac{2x^2+x-12}{x(x+3)(x+2)}$. Find constants by comparing coefficients.

Answers:

a) $$\frac{2x^2+x-12}{x(x+3)(x+2)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x+2}$$

$$\Rightarrow 2x^2+x-12 = A(x+3)(x+2) + Bx(x+2) + Cx(x+3)$$
$$\Rightarrow 2x^2+x-12 = A(x^2+5x+6) + B(x^2+2x) + C(x^2+3x)$$
$$\Rightarrow 2x^2+x-12 = x^2(A+B+C) + x(5A+2B+3C) + 6A$$
$$\Rightarrow A+B+C = 2,\ 5A+2B+3C = 1 \text{ and } 6A = -12 \text{ or } A = -2.$$

After substituting $A = -2$ into the first two equations:

$$\left.\begin{aligned} B + C &= 4 \\ 2B + 3C &= 11 \end{aligned}\right\} \Rightarrow C = 3 \text{ and } B = 1.$$

As a result: $\dfrac{2x^2+x-12}{x(x+3)(x+2)} = \dfrac{-2}{x} + \dfrac{1}{x+3} + \dfrac{3}{x+2}$.

b) Answer on next page.

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b) Decompose: $\dfrac{4x^2+2x-1}{x^3+x^2}$. Find constants by comparing coefficients.

Answer:

b) $$\frac{4x^2+2x-1}{x^3+x^2} = \frac{4x^2+2x-1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$\Rightarrow 4x^2+2x-1 = Ax(x+1) + B(x+1) + Cx^2$$
$$\Rightarrow 4x^2+2x-1 = (Ax^2+Cx^2) + (Ax+Bx) + B$$
$$\Rightarrow 4x^2+2x-1 = (A+C)x^2 + (A+B)x + B$$

After comparing coefficients:

$$A+C=4, \qquad A+B=2, \qquad B=-1.$$

As a result:

$$A=3, \qquad B=-1, \qquad C=1$$

and $$\frac{4x^2+2x-1}{x^3+x^2} = \frac{3}{x} + \frac{-1}{x^2} + \frac{1}{x+1}$$

Can you think of a problem where this will prove helpful? What about $\int \dfrac{4x^2+2x-1}{x^3+x^2}\,dx$?

Return to Review Topic

Which of the following is irreducible?

a) $x^2 - 10$ b) $x^2 + 9$ c) $x^2 + 2x + 4$

Answers:

a) $b^2 - 4ac = 40 > 0$; reduces into, $(x - \sqrt{10})(x + \sqrt{10})$

b) $b^2 - 4ac = -36 < 0$; irreducible.

c) $b^2 - 4ac = -12 < 0$; irreducible.

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Set up the decomposition for $\dfrac{x^2+9}{x^4-2x^2-8}$.

Answer:

$$\frac{x^2+9}{x^4-2x^2-8} = \frac{x^2+9}{(x-2)(x+2)(x^2+2)} = \frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+2}$$

Have you noticed that in every term of the decomposition, the numerator is always one less in degree than it's denominator?

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Decompose $\dfrac{2x-1}{x(x^2+4)}$

Answer:

$$\begin{aligned}
\frac{2x-1}{x(x^2+4)} &= \frac{A}{x} + \frac{Bx+C}{x^2+4} \\
2x-1 &= A(x^2+4) + (Bx+C)x \\
2x-1 &= (A+B)x^2 + Cx + 4A \\
0x^2+2x-1 &= (A+B)x^2 + Cx + 4A
\end{aligned}$$

Comparing coefficients

$$-1 = 4A, \quad C = 2, \quad A+B=0 \Rightarrow A = -\frac{1}{4},\ B = \frac{1}{4},\ C = 2$$

Decomposition: $\dfrac{2x-1}{x(x^2+4)} = \dfrac{-\frac{1}{4}}{x} + \dfrac{-\frac{1}{4}x+2}{x^2+4}$

Note: A 'blending' of both techniques is also possible. After doing more problems you'll discover what works best.

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a) $x^3 + 3x^2 - 2x - 6$

b) $x^4 - x$

c) $x^3 - x + 6$

Answers:

a)
$$\begin{aligned} x^3 + 3x^2 - 2x - 6 &= x^2(x+3) - 2(x+3) \\ &= (x^2 - 2)(x+3) \\ &= (x - \sqrt{2})(x + \sqrt{2})(x+3) \end{aligned}$$

b) $x^4 - x = x(x^3 - 1) = x(x-1)(x^2 + x + 1)$ $\left(\begin{array}{c} b^2 - 4ac = -3 \\ \text{irreducible} \end{array}\right)$

c) Since $p(-2) = 0$, $x + 2$ is a factor.

$$x + 2 \overline{\big)\, x^3 - x - 6} = x^2 - 2x + 3$$

so $x^3 - x + 6 = (x+2)(x^2 - 2x + 3)$

$\nwarrow$ irreducible, $b^2 - 4ac = -8$

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