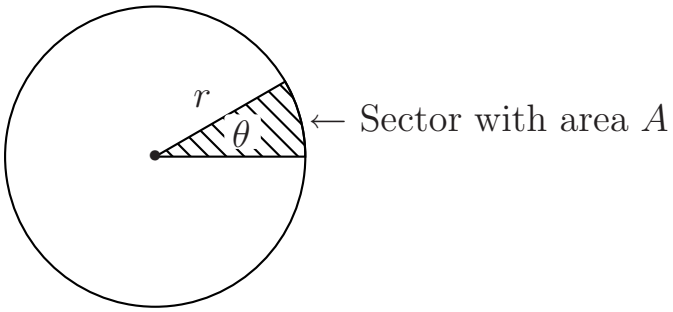
MATH 250 – REVIEW TOPIC 6

Area of a Sector of a Circle

A sector of a circle looks like a piece of pie.

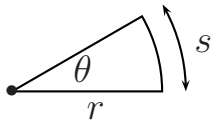


In Calc II, it is important to know the formula for finding the area, A, of a sector of a circle of radius r and angle θ (in radians). Let's derive it.

The circle itself has an area equal to πr^2 and covers an angle of 2π. Using proportions, the area of the circle is to the angle 2π as the area of the sector A is to θ. That is,

$$\frac{\pi r^2}{2\pi} = \frac{A}{\theta}, \text{ or}$$
$$A = \frac{1}{2}r^2\theta.$$

We can also write the area A of the sector in terms of the arc length. The arc length, s, subtended by θ is given by the formula $s = r\theta$ (see Math 150, Review Topic 10).



$s = r\theta$

Thus, we have

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(r\theta) = \frac{1}{2}rs.$$

(The formula $A = \frac{1}{2}rs$ reminds us of the formula for the area of a triangle, $\frac{1}{2}bh$, where r is "like" the base b and s is "like" the height h.)

Exercise 1: Let $r = 3$ and $\theta = \dfrac{3\pi}{4}$. Find the area of the sector of the circle.

Solution: $A = \dfrac{1}{2}r^2\theta$. If $r = 3$ and $\theta = \dfrac{3\pi}{4}$, then $A = \dfrac{1}{2}(3)^2 \cdot \left(\dfrac{3\pi}{4}\right) = \dfrac{27\pi}{8}$.

Beginning of Topic Review Topics 250 Skills Assessment