

# COLLOQUIUM

Lakshika Gunawardana

Department of Mathematics,  
SOUTHERN ILLINOIS UNIVERSITY

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## Locally Primitively Universal forms and the Primitive Counterpart of the Fifteen Theorem

### [ABSTRACT]

The systematic study of positive definite integral quadratic forms that represent all positive integers (or all sufficiently large positive integers) was initiated by Ramanujan a little over a century ago. Such forms are now referred to as universal (or almost universal) forms. In a groundbreaking 1917 paper, Ramanujan determined all forms of the type  $ax^2 + by^2 + cz^2 + du^2$  that are universal, and all those of the special type  $a(x^2 + y^2 + z^2) + du^2$  that are almost universal.

In 1993, J.H. Conway and W.A. Schneeberger presented the Fifteen Theorem, which provides simple criteria to determine whether a positive definite classically integral quadratic form in any number of variables is universal. Later in 2000, M. Bhargava provided a refinement of the Fifteen Theorem and showed that there are exactly 204 positive definite classically integral quaternary quadratic forms, up to equivalence, which are universal. We try to determine which of the forms in the 204 list are primitively universal, and try to determine whether there exists a finite set  $S$  of integers such that every positive definite integral quadratic form that primitively represents the integers in  $S$ , primitively represents all positive integers. In the first half of this talk, we introduce quadratic forms in general with a brief history and present a conjecture which could be a primitive counterpart to the Fifteen Theorem. Then we review the  $p$ -adic numbers and  $p$ -adic norm and discuss their application of almost universal quadratic forms, concluding with some new results on almost primitively universal forms.

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Speaker's Website: <https://math.siu.edu/faculty-staff/grad-stud/phd-stud/gunawardana.php>