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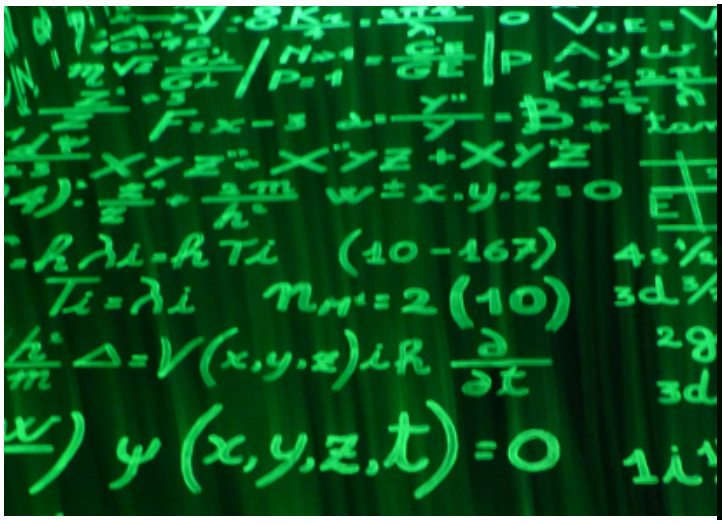

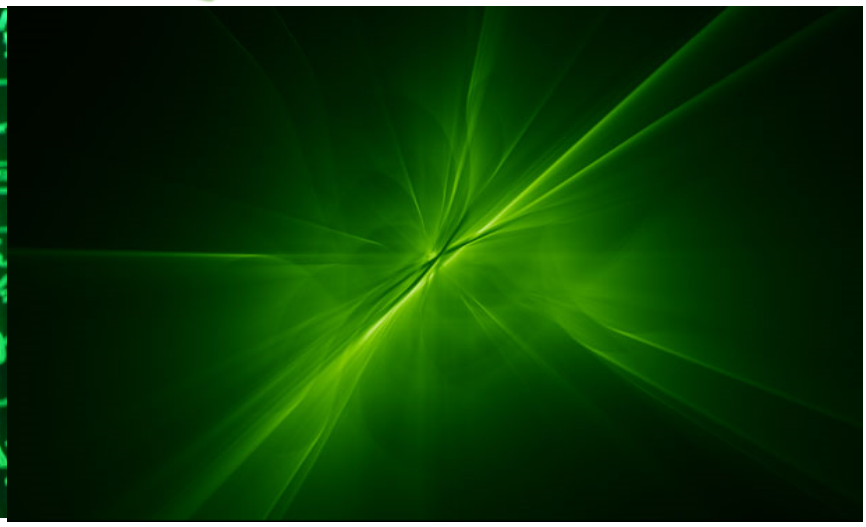

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Abstract:

INFINITELY MANY SIGN-CHANGING SOLUTIONS TO A CONFORMALLY INVARIANT INTEGRAL EQUATION

MATHEW GLUCK

ABSTRACT

In $n \geq 3$ dimensions, the classical Sobolev inequality asserts the existence of a positive lower bound for the functional

$$Q(u) = \frac{\int_{\mathbb{R}^n} |\nabla u|^2 \, dx}{\left(\int_{\mathbb{R}^n} |u|^{\frac{2n}{n-2}} \, dx \right)^{(n-2)/n}}$$

as u varies through a suitable class of functions. The analysis of this functional has a variety of applications in both pure an applied mathematics. Of particular importance is the *extremal problem* for Q which seeks to determine both the minimal value of Q and the functions u for which the minimal value is attained. If one is interested in understanding the minimizers of Q, one is naturally led to analyze the Euler-Lagrange equation $Q'(u) = 0$. A simple computation shows that up to multiplication by -1, the minimizers of Q must be strictly positive on $\mathbb{R}^n$. Moreover, the classical results of Gidas, Ni, Nirenberg and Cafarelli, Gidas, Spruck show that modulo the action of the conformal group of $\mathbb{R}^n$, the positive solutions of the Euler-Lagrange equation (and in particular the minimizers of Q) must all be of the form

$$U(x) = (1 + |x|^2)^{-(n-2)/2}.$$

This classification result inspired many attempts to show that up to a conformal transformation of $\mathbb{R}^n$, any solution of the Euler-Lagrange equation that is positive *somewhere* must also be of the above form. An influential result of Ding showed that these attempts were destined to fail.

In this talk I will discuss some recent work of mine that is inspired by Ding's result. In short, my work establishes a result similar in spirit to that of Ding, but for the Hardy-Littlewood-Sobolev functional, a functional that can be viewed as a dual functional to Q. To contextualize my work, I will overview the history and applications of the study of the critical points of Q. In addition, I will discuss both the underlying idea behind Ding's argument and a variety of extensions of Ding's result based on this idea.

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