COLLOQUIUM SERIES



Structure & Complexity of Orders on Structures

[ABSTRACT]

We study topological and computability-theoretic properties of orders on magmas. A magma is a structure with a binary operation. It is right-orderable if there is a linear ordering of its domain, which is right-invariant with respect to the operation. If the ordering is also left-invariant, then a magma is bi-orderable. Interesting rightorderable magmas the operation of which is self-distributive and not necessarily associative come from knot theory and are called quandles. There is a natural topology on the set of all right (left) orders or bi-orders of orderable magmas. These spaces are compact. A computable orderable group does not necessarily have a computable order. This can be used to show that the space of orders on such a group is homeomorphic to the Cantor set. For a large class of orderable residually nilpotent groups, including free groups and surface groups, we obtain orders in all Turing degrees.

