Part I. Problems in this section are mostly short answer and multiple choice. Little partial credit will be given. 5 points each.

1. Factor **completely**. Do not solve.

a) $2x^2 + 20x + 18$

$2(x^2+10x+9)$

$\boxed{2(x+9)(x+1)}$

b) $x^3 - 2x^2 + 3x - 6$

$x^2(x-2)+3(x-2)$

$\boxed{(x^2+3)(x-2)}$

2. Find the quotient and remainder.

$(x^3 - 3x + 1) \div (x + 2)$

Quotient: x^2-2x+1

Remainder: -1

-2	1	0	-3	1
		-2	4	-2
	1	-2	1	-1

x^2-2x+1 Remainder

3. State the slope and y-intercept for the line $5x + 2y = 6$

$-5x \qquad -5x$

$\frac{2y}{2} = \frac{-5x+6}{2}$

$y = \frac{-5}{2}x + 3$

$m =$ $-\frac{5}{2}$

$y-$int: $(0,3)$

(write it as an ordered pair)

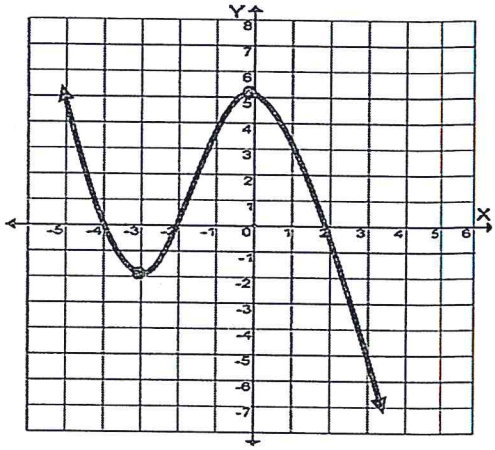
4. Given the graph of $f(x)$, state all x such that:

a) $f(x)$ is increasing (use interval notation)

$(-3, 0)$

b) $f(x) < 0$ (use interval notation)

$(-4, -2) \cup (2, \infty)$



5. Solve: $4 \cdot -3 \le \frac{2+x}{4} \cdot 4 < 6 \cdot 4$. Express your answer in interval notation.

$-12 \le 2 + x < 6$

$-2 \quad -2 \quad -2$

$-14 \le x < 4$

$[-14, 4)$

6. Find the domain of the function $g(x) = \sqrt{x+3}$. Circle the correct answer.

a) $(-\infty, \infty)$
b) $(-3, \infty)$
c) $(-\infty, -3)$
d) $[-3, \infty)$
e) $(-\infty, -3]$

$x + 3 \ge 0$

$x \ge -3$

$[-3, \infty)$

7. Solve for x. Show all work and circle your final answer.

a) $x^2 - 5x = 6$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x - 6 = 0 \quad x + 1 = 0$

$+6 \quad +6 \quad -1 \quad -1$

$\boxed{x = 6} \quad \boxed{x = -1}$

b) $4 \cdot A = \frac{1}{4} \cdot 4 xy$

$\frac{4A}{y} = \frac{xy}{y}$

$\frac{4A}{y} = x \rightarrow \boxed{x = \frac{4A}{y}}$

8. Let $f(x) = 5x - 3$ and $g(x) = x^2 + 1$. Find and simplify.

a) $(f \circ g)(2) = f(g(2))$

$g(2) = (2)^2 + 1$
$= 4 + 1$
$= 5$

$= f(5)$
$= 5(5) - 3$
$= 25 - 3$
$= \boxed{22}$

b) $(f - g)(x) = f(x) - g(x)$
$= 5x - 3 - (x^2 + 1)$
$= \boxed{-x^2 + 5x - 4}$

9. Solve ALGEBRICALLY. Show all work. $|2x - 3| + 4 = 11$

$-4 \quad -4$

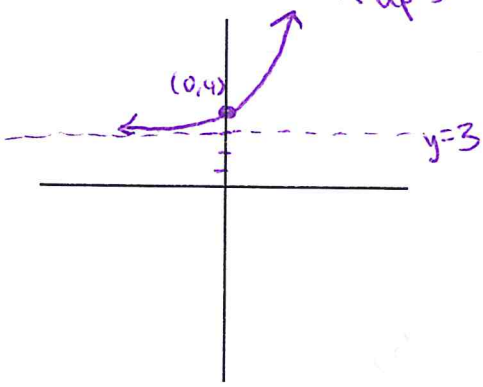
$|2x - 3| = 7$

$2x - 3 = 7$
$+3 \quad +3$
$\frac{2x}{2} = \frac{10}{2}$
$\boxed{x = 5}$

$2x - 3 = -7$
$+3 \quad +3$
$\frac{2x}{2} = \frac{-4}{2}$
$\boxed{x = -2}$

10. Graph each function. Dash in asymptotes where needed. Label all intercepts and asymptotes!

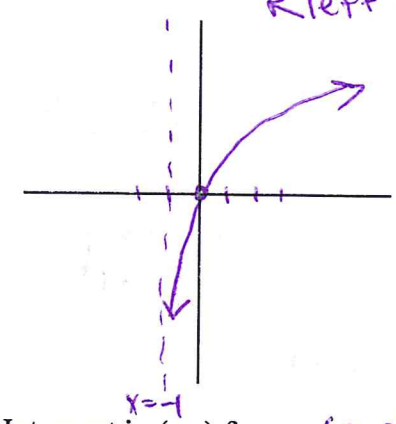
$f(x) = e^x + 3$ ↖ up 3



Intercept in (x,y) form: (0,4)

Equation of asymptote: y=3

$g(x) = \ln(x+1)$ ↖ left 1



Intercept in (x,y) form: (0,0)

Equation of asymptote: x = -1

11. Find the product. Express in $a + bi$ form. $(4 + 6i)(1 - 3i)$

$4 - 12i + 6i - 18i^2$

$4 - 6i - 18(-1)$

$4 - 6i + 18$

$\boxed{22 - 6i}$

12. Find the <u>slope</u> of linear function f such that $f(3) = 2$ and $f(0) = -1$.

$(3, 2)$ $(0, -1)$

$m = \frac{-1-2}{0-3} = \frac{-3}{-3} = 1$

$m =$ 1

13. Write a polynomial of degree 3 that has zeros: 2 and $4i$. Write final answer in polynomial form (multiplied out).

zeros: $x = 2$ $x = 4i$ $x = -4i$

factors: $(x-2)(x-4i)(x+4i)$

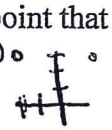
$(x-2)(x^2 + \cancel{4ix} - \cancel{4ix} - 16i^2)$

$(x-2)(x^2 + 16)$

$f(x) =$ $x^3 - 2x^2 + 16x - 32$ $x^3 - 2x^2 + 16x - 32$

14. Given the point $(-2, 3)$, find a point that is symmetric to the given point:

a) with respect to the y-axis.



b) with respect to the origin.

$(2, 3)$

$(2, -3)$

Part II. There are 9 problems in this section. Show all work. 10 points each.

15. A stone is thrown directly upward. Its height after t seconds is given by the function $h(t) = -3t^2 + 6t + 4$. The height of the stone is measured in feet. Show your work algebraically and include units on your answers.

a) What is the initial height of the stone?

$t=0$

$h(0) = \boxed{4 \text{ ft.}}$

b) How long does it take for the stone to reach its maximum height?

$t=?$

time height

$\hookrightarrow$ vertex: $\left(\frac{-b}{2a}, h\left(\frac{-b}{2a}\right)\right)$

$t = \frac{-b}{2a} = \frac{-6}{2(-3)} = \frac{-6}{-6} = \boxed{1 \text{ second.}}$

c) What is the maximum height the stone reaches?

vertex:

$h(1) = -3(1)^2 + 6(1) + 4$
$= -3 + 6 + 4$
$= \boxed{7 \text{ ft.}}$

16. Find all asymptotes, ~~x-intercepts~~ (zeros), and y-intercepts for the graph $f(x) = \frac{4x-6}{x-2}$.

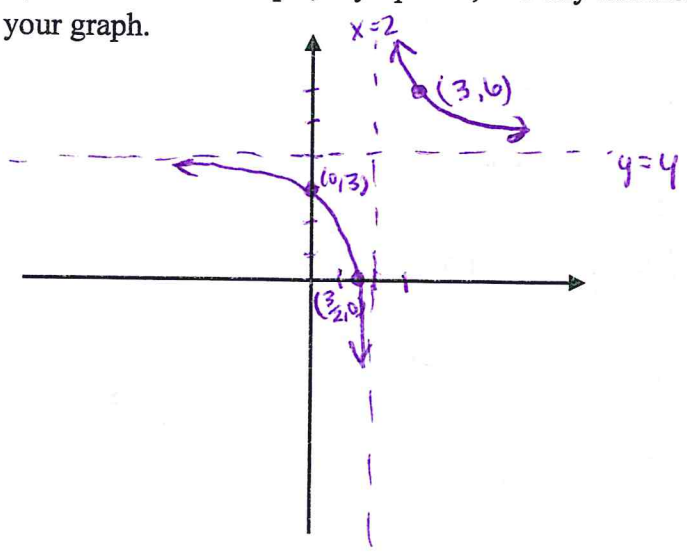
a) The equation of the vertical asymptote(s) is/are $x =$ 2.

b) The equation of the horizontal asymptote(s) is/are $y =$ 4.

c) The x-intercept (or zero) is at the point $\left(\frac{3}{2}, 0\right)$. $4x - 6 = 0 \Rightarrow 4x = 6 \Rightarrow x = \frac{6}{4} = \frac{3}{2}$

d) The y-intercept is at the point $(0, 3)$. $f(0) = \frac{4(0)-6}{0-2} = \frac{-6}{-2} = 3$

e) Sketch the graph of $f(x)$. Label all intercepts, asymptotes, and any additional points you found to help improve your graph.



X	y
3	$\frac{4(3)-6}{3-2} = \frac{12-6}{1} = 6$

17. Find a formula for the inverse given $f(x) = \frac{x+1}{3x-2}$.

$$y = \frac{x+1}{3x-2}$$

$$(3y-2)x = \frac{y+1}{3y-2} \cdot (3y-2)$$

$$3xy - 2x = y + 1$$

$$-y \quad +2x \quad -y \quad +2x$$

$$3xy - y = 2x + 1 \longrightarrow y\frac{(3x-1)}{3x-1} = \frac{2x+1}{3x-1}$$

$$y = \frac{2x+1}{3x-1}$$

$$\boxed{f^{-1}(x) = \frac{2x+1}{3x-1}}$$

18. Solve algebraically for x.

a) $4^{2x} = 8^{3x-1}$

$$(2^2)^{2x} = (2^3)^{(3x-1)}$$

$$2^{4x} = 2^{9x-3}$$

$$4x = 9x - 3$$

$$-5x = -3$$

$$\boxed{x = \frac{3}{5}}$$

b) $\ln(5x - 9) = 0$

$$e^0 = 5x - 9$$

$$1 = 5x - 9$$

$$\frac{10}{5} = \frac{5x}{5}$$

$$2 = x \rightarrow \boxed{x = 2}$$

19. Suppose \$600 is invested in a savings account in which interest is compounded continuously at 2% per year. The amount of money in the account t years later is given by the equation: $A = 600e^{0.02t}$. Find the amount of time it would take the amount to reach \$2000. Leave your answer in exact form since no calculators are allowed.

$$\frac{2000}{600} = \frac{600e^{0.02t}}{600}$$

$$\frac{10}{3} = e^{0.02t}$$

$$\ln\left(\frac{10}{3}\right) = \ln\left(e^{0.02t}\right)$$

$$\ln\left(\frac{10}{3}\right) = 0.02t$$

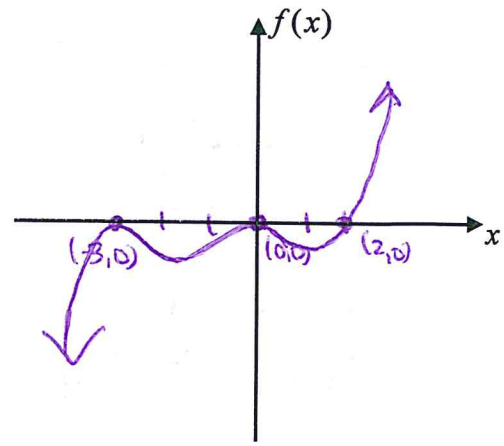
$$\frac{\ln\left(\frac{10}{3}\right)}{0.02} = t \longrightarrow \boxed{t = \frac{\ln\left(\frac{10}{3}\right)}{0.02} \text{ years}}$$

20. Given the function $f(x) = x^2(x-2)(x+3)^2$,

a) Find the y-intercept.

$f(0) = (0)^2(0-2)(0+3)^2$
$= 0 \cdot (-2)(9) = 0$

$\boxed{(0,0)}$



b) Find all zeros and state their multiplicities.

zero	multiplicity
0	2
2	1
-3	2

c) Is $f(x)$ tangent to the x-axis? Yes
If so, where?

at $x=0$ and $x=-3$

d) Draw ending behavior. $x^{2}(x^{1}-2)(x+3)^{2}$ → degree 5 ← odd
and positive leading coefficient

e) Sketch graph. Label all intercepts.

21. Solve algebraically: $(\sqrt{6x+7})^2 = (x+2)^2$ Check all solutions.

$6x+7 = (x+2)(x+2)$

$6x+7 = x^2+4x+4$
$-6x \quad -7 \qquad -6x \quad -7$

$0 = x^2-2x-3$

$0 = (x-3)(x+1)$

$x-3=0 \qquad x+1=0$

$\boxed{x=3 \qquad x=-1}$

Check:
$x=3$: $\sqrt{6(3)+7} = 3+2$
$\sqrt{25} = 5$ ✓

$x=-1$ $\sqrt{6(-1)+7} = -1+2$
$\sqrt{1} = 1$ ✓

22. Given the function $f(x) = x^2 + 6x + 5$

a) State the y-intercept.

$f(0) = 0^2 + 6(0) + 5 = 5$

$\boxed{(0,5)}$

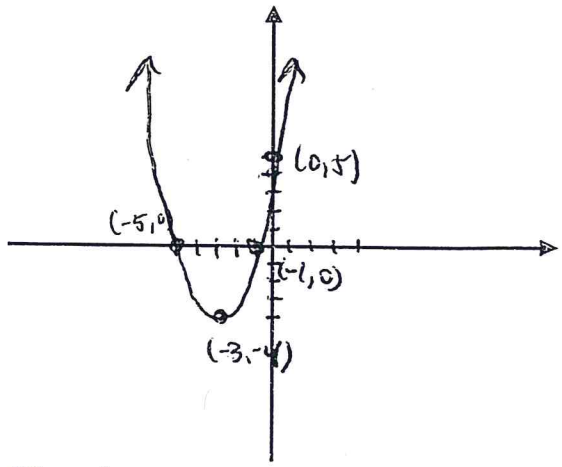
b) State the zeros of the function.

$x^2 + 6x + 5 = 0$

$(x+5)(x+1) = 0$

$x+5=0 \quad x+1=0$

$\boxed{x=-5} \quad \boxed{x=-1}$



c) The vertex is (-3 , -4).

$\frac{-b}{2a} = \frac{-6}{2(1)} = -3$

$f(-3) = (-3)^2 + 6(-3) + 5 = 9 - 18 + 5 = -4$

d) State the range of f.

$[-4, \infty)$

e) Graph. Label intercepts and vertex.

23. Given the polynomial $g(x) = x^3 - x^2 + x - 6$

1 $\quad$ 1,2,3,6 $\div$ 1

a) State all possible rational zeros.

$\boxed{\pm\{1,2,3,6\}}$

b) Find all zeros (real and complex.)

1	1	-1	1	-6
		1	1	2
	1	0	2	-4

not zero

-1	1	-1	1	-6
		-1	2	-3
	1	-2	3	-9

2	1	-1	1	-6
		2	2	6
	1	1	3	0

← zero, so $x=2$ is a zero

$x^2 + x + 3 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(3)}}{2(1)} = \frac{-1 \pm \sqrt{-11}}{2} = \frac{-1 \pm i\sqrt{11}}{2}$

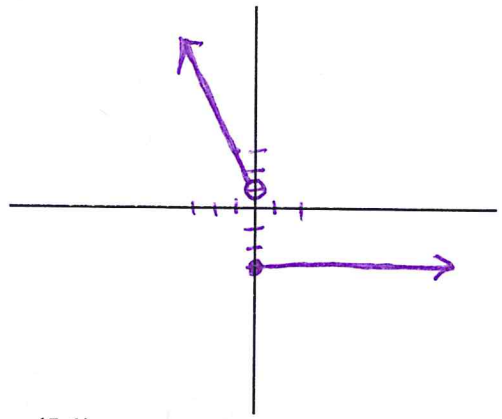
Answer: $x = 2 \quad x = \frac{-1+i\sqrt{11}}{2} \quad x = \frac{-1-i\sqrt{11}}{2}$

Part III. There are 6 problems in this section. Choose any 4. Indicate in the boxes the problems you want graded. 10 points each. If you do not indicate which 4, the first 4 will be graded. No Extra Credit!

☐ Grade

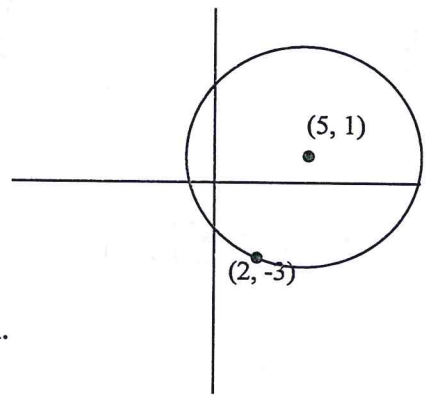
24. Graph the following function.

$$f(x)=\begin{cases}-2x+1, & x<0\\ -3 & ,x\geq 0\end{cases}$$



☐ Grade

25. The graph to the right is a circle with center (5,1).



a) Find the length of the radius.

r = distance between (5,1) & (2,-3)

$= \sqrt{(5-2)^2+(1-(-3))^2}$

$= \sqrt{3^2+4^2}$

$= \sqrt{9+16} = \sqrt{25} = \boxed{5}$

b) State the equation of the circle in standard form.

$$\boxed{(x-5)^2+(y-1)^2=25}$$

☐ Grade

26. Given $f(x) = 2x^2 + 4x$, find and simplify $\dfrac{f(x+h)-f(x)}{h}$.

note: $f(x+h) = 2(x+h)^2 + 4(x+h)$

$= 2(x^2+2xh+h^2) + 4x + 4h$

$= 2x^2 + 4xh + 2h^2 + 4x + 4h$

$$\frac{f(x+h)-f(x)}{h} = \frac{\cancel{2x^2}+4xh+2h^2+\cancel{4x}+4h-(\cancel{2x^2}+\cancel{4x})}{h}$$

$$= \frac{\cancel{h}(4x+2h+4)}{\cancel{h}} = \boxed{4x+2h+4}$$

☐ Grade

27. Solve algebraically for x. $\log_2 x + \log_2(x-2) = 3$

$\log_2(x(x-2)) = 3$

$2^3 = x^2 - 2x$

$8 = x^2 - 2x$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2) \Rightarrow \boxed{x=4}$ $\cancel{x=-2}$

Check:

$x=4$ $\log_2 4 + \log_2(2) = 3$

$2 + 1 = 3$ ✓

$x=-2$ $\log_2(-2)$

can't do!

☐ Grade

28. Solve $\frac{x+2}{x-3} \geq 0$. Express in interval form. To receive full credit you must show work that supports your answer.

VA: $x-3=0$

$x=3$

zeros: $x+2=0$

$x=-2$

$(-\infty,-2]$ $\cancel{[-2,3)}$ $(3,\infty)$

-2 3

$x=-3$ $x=0$ $x=4$

Test values:

$x=-3$ $\frac{-3+2}{-3-3} = \frac{-1}{-6} = \frac{1}{6} \geq 0$ ✓ True

$x=0$ $\frac{0+2}{0-3} = -\frac{2}{3} \not\geq 0$ ✗ False

$x=4$ $\frac{4+2}{4-3} = \frac{6}{1} = 6 \geq 0$ ✓ True

$(-\infty,-2] \cup (3,\infty)$

☐ Grade

29. Consider the following system: $\begin{cases} 2x - 3y = -6 \\ x + y = 2 \end{cases}$

a) Solve algebraically
Show all your work.

$2x - 3y = -6$

$3(x + y = 2) \rightarrow 3x + 3y = 6$

$2x - 3y = -6$

$5x = 0$

$x = 0$

$y = 2$

b) Solve graphically and explain how you obtained your answer by looking at the graph.

$2x - 3y = -6$

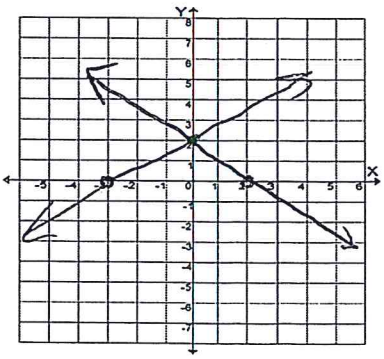
x-int: $(-3,0)$

y-int: $(0,2)$

$x + y = 2$

x-int: $(2,0)$

y-int: $(0,2)$



$(0,2)$ point where the graphs intersect

Answer: $(0,2)$