Part I. Problems in this section are mostly short answer and multiple choice. Little partial credit will be given. 5 points each.

1. Factor completely. Do not solve.
   
   a) \(2x^2 + 20x + 18\)
   
   \[
   \frac{2(x^2 + 10x + 9)}{2(x+9)(x+1)}
   \]

   b) \(x^3 - 2x^2 + 3x - 6\)

   \[
   \frac{x^2(x-2) + 3(x-2)}{(x^2+3)(x-2)}
   \]

2. Find the quotient and remainder.

   
   \((x^3 - 3x + 1) + (x + 2)\)

   \[
   \begin{array}{c|cccc}
   -2 & 1 & 0 & -3 & 1 \\
   \hline
   & -2 & 4 & -2 & \frac{1}{1} \\
   \end{array}
   \]

   Quotient: \(x^2 - 2x + 1\)

   Remainder: \(-1\)

3. State the slope and y-intercept for the line \(5x + 2y = 6\)

   \[
   2y = -5x + 6
   \]

   \[
   y = -\frac{5}{2}x + 3
   \]

   \[
   m = -\frac{5}{2} \quad \text{y - int: } (0, 3)
   \]

   (write it as an ordered pair)
4. Given the graph of \( f(x) \), state all \( x \) such that:

   a) \( f(x) \) is increasing (use interval notation)
      \[ (-3, 0) \]

   b) \( f(x) < 0 \) (use interval notation)
      \[ (-4, -2) \cup (2, \infty) \]

5. Solve: \[ \frac{4}{3} \leq \frac{2 + x}{4} < 6 \]. Express your answer in interval notation.

   \[ -12 \leq 2 + x < 24 \]
   \[ -14 \leq x < 4 \]
   \[ [-14, 4] \]

6. Find the domain of the function \( g(x) = \sqrt{x + 3} \). Circle the correct answer.
   a) \( (-\infty, \infty) \)
   b) \( (-3, \infty) \)
   c) \( (-\infty, -3) \)
   d) \( [-3, \infty) \)
   e) \( (-\infty, -3] \)

7. Solve for \( x \). Show all work and circle your final answer.
   a) \( x^2 - 5x = 6 \)
      \[ x^2 - 5x - 6 = 0 \]
      \[ (x - 6)(x + 1) = 0 \]
      \[ x - 6 = 0 \quad x + 1 = 0 \]
      \[ x = 6 \quad x = -1 \]

   b) \( \frac{4A}{x} = \frac{xy}{5} \)
      \[ \frac{4A}{y} = x \rightarrow x = \frac{4A}{y} \)
8. Let \( f(x) = 5x - 3 \) and \( g(x) = x^2 + 1 \). Find and simplify.

\[ \text{a)} \quad (f \circ g)(2) = f(g(2)) \]
\[ g(2) = (2)^2 + 1 = 5 \]
\[ f(5) = 5(5) - 3 = 25 - 3 = 22 \]

\[ \text{b)} \quad (f - g)(x) = f(x) - g(x) \]
\[ = 5x - 3 - (x^2 + 1) \]
\[ = -x^2 + 5x - 4 \]

9. Solve ALGEBRICALLY. Show all work. \(|2x - 3| + 4 = 11\)
\[
\begin{align*}
\left| 2x - 3 \right| &= 7 \\
2x - 3 &= 7 \\
2x &= 10 \\
x &= 5 \\
\text{or} \\
2x - 3 &= -7 \\
2x &= -4 \\
x &= -2
\end{align*}
\]

10. Graph each function. Dash in asymptotes where needed. **Label all intercepts and asymptotes!**

\[ f(x) = e^x + 3 \]
Intercept in \((x, y)\) form: \((0, 4)\)
Equation of asymptote: \(y = 3\)

\[ g(x) = \ln(x + 1) \]
Intercept in \((x, y)\) form: \((0, 0)\)
Equation of asymptote: \(x = -1\)
11. Find the product. Express in $a + bi$ form. \((4 + 6i)(1 - 3i)\)
\[
\begin{align*}
4 - 12i + 6i - 18i^2 \\
4 - 6i - 18(-1) \\
4 - 6i + 18 \\
22 - 6i
\end{align*}
\]

12. Find the slope of linear function $f$ such that $f(3) = 2$ and $f(0) = -1$.
\[
(3, 2) \quad (0, -1)
\]
\[
m = \frac{2 - (-1)}{3 - 0} = \frac{3}{3} = 1
\]

13. Write a polynomial of degree 3 that has zeros: 2 and $4i$. Write final answer in polynomial form (multiplied out).
\[
\text{Zeros: } x = 2 \quad x = 4i \quad x = -4i
\]
\[
\text{Factors: } (x - 2)(x - 4i)(x + 4i)
\]
\[
\begin{align*}
(x - 2) & (x^2 + 16) \\
(x^2 - 4i)(x - 4i) & (x^2 + 16)
\end{align*}
\]
\[
f(x) = x^3 - 2x^2 + 16x - 32 \quad x^3 - 2x^2 + 16x - 32
\]

14. Given the point $(-2, 3)$, find a point that is symmetric to the given point:

a) with respect to the $y$-axis.
\[
(-2, 3)
\]

b) with respect to the origin.
\[
(2, -3)
\]
Part II. There are 9 problems in this section. Show all work. 10 points each.

15. A stone is thrown directly upward. Its height after \( t \) seconds is given by the function \( h(t) = -3t^2 + 6t + 4 \). The height of the stone is measured in feet. Show your work algebraically and include units on your answers.

a) What is the initial height of the stone?

\[ h(0) = \square \quad \text{feet} \]

b) How long does it take for the stone to reach its maximum height?

\[ t = \frac{-b}{2a} = \frac{-6}{2(-3)} = \frac{-6}{-6} = 1 \text{ second} \]

vertex: \( (-\frac{b}{2a}, h(-\frac{b}{2a})) \)

c) What is the maximum height the stone reaches?

\[ h(1) = -3(1)^2 + 6(1) + 4 = -3 + 6 + 4 = 7 \text{ feet} \]

16. Find all asymptotes, \( x \)-intercepts (zeros), and \( y \)-intercepts for the graph \( f(x) = \frac{4x-6}{x-2} \).

a) The equation of the vertical asymptote(s) is/are \( x = 2 \).

b) The equation of the horizontal asymptote(s) is/are \( y = 4 \).

c) The \( x \)-intercept (or zero) is at the point \( \left( \frac{3}{2}, 0 \right) \). \( 4x-6=0 \Rightarrow 4x=6 \Rightarrow x=\frac{6}{4}=\frac{3}{2} \)

d) The \( y \)-intercept is at the point \( (0,3) \). \( f(0)=\frac{4(0)-6}{0-2} = \frac{-6}{-2} = 3 \)

e) Sketch the graph of \( f(x) \). Label all intercepts, asymptotes, and any additional points you found to help improve your graph.
17. Find a formula for the inverse given \( f(x) = \frac{x+1}{3x-2} \)

\[ y = \frac{x+1}{3x-2} \]

\[ (3y-2)x = \frac{y+1}{3y-2} \]

\[ 3xy - 2x = \frac{y+1}{3y-2} \]

\[ 3xy - y = 2x + 1 \]

\[ \frac{y(3x-1)}{3x-1} = \frac{2x+1}{3x-1} \]

\[ y = \frac{2x+1}{3x-1} \]

\[ f^{-1}(x) = \frac{2x+1}{3x-1} \]

18. Solve algebraically for \( x \).

a) \( 4^{2x} = 8^{3x-1} \)

\[
(\frac{2^2}{2^3})^{2x} = (\frac{2^3}{2^3})^{3x-1}
\]

\[ 2^{4x} = 2^{9x-3} \]

\[ 4x = 9x - 3 \]

\[ -5x = -3 \]

\[ x = \frac{3}{5} \]

b) \( \ln(5x - 9) = 0 \)

\[ e^0 = 5x - 9 \]

\[ 1 = 5x - 9 \]

\[ 10 = 5x \]

\[ \frac{10}{5} = \frac{x}{5} \]

\[ x = 2 \]

19. Suppose $600$ is invested in a savings account in which interest is compounded continuously at $2\%$ per year. The amount of money in the account \( t \) years later is given by the equation: \( A = 600e^{0.02t} \). Find the amount of time it would take the amount to reach $2000$. Leave your answer in exact form since no calculators are allowed.

\[
\frac{2000}{600} = \frac{600e^{0.02t}}{600}
\]

\[ \frac{10}{3} = e^{0.02t} \]

\[ \ln\left(\frac{10}{3}\right) = \ln\left(e^{0.02t}\right) \]

\[ \ln\left(\frac{10}{3}\right) = 0.02t \]

\[ \frac{\ln\left(\frac{10}{3}\right)}{0.02} = t \]

\[ t = \frac{\ln\left(\frac{10}{3}\right)}{0.02} \text{ years} \]
20. Given the function \( f(x) = x^2(x - 2)(x + 3)^2 \),

a) Find the y-intercept.
\[
f(0) = (0)^2 (0 - 2)(0 + 3)^2 = 0 \\
\text{y-intercept: } (0, 0)
\]

b) Find all zeros and state their multiplicities.

<table>
<thead>
<tr>
<th>zero</th>
<th>multiplicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>-3</td>
<td>2</td>
</tr>
</tbody>
</table>

c) Is \( f(x) \) tangent to the x-axis? Yes
If so, where?
\[ x = 0 \text{ and } x = -3 \]

d) Draw ending behavior.
\[
x^2(x - 2)(x + 3)^2 \rightarrow \text{degre 5 < odd} \\
\text{and positive leading coefficient}
\]

e) Sketch graph. Label all intercepts.

21. Solve algebraically: \( \sqrt{6x+7} = (x+2)^2 \) Check all solutions.

\[
\begin{align*}
6x + 7 &= (x + 2)(x + 2) \\
0 &= x^2 + 4x + 4 \\
&-6x - 7 \\
&-6x - 7 \\
0 &= x^2 - 2x - 3 \\
0 &= (x - 3)(x + 1) \\
x = 3 &\quad \text{or} \quad x = -1 \\
\text{Check:} \\
x = 3: \sqrt{6(3)+7} = 3 + 2 \\
\sqrt{25} = 5 \\
x = -1: \sqrt{6(-1)+7} = -1 + 2 \\
\sqrt{1} = 1 \\
\end{align*}
\]
22. Given the function \( f(x) = x^2 + 6x + 5 \)
   
   a) State the y-intercept.
   \[ f(0) = 0^2 + 6(0) + 5 = 5 \]
   \[ (0, 5) \]
   
   b) State the zeros of the function.
   \[ x^2 + 6x + 5 = 0 \]
   \[ (x + 5)(x + 1) = 0 \]
   \[ x + 5 = 0 \quad x + 1 = 0 \]
   \[ x = -5 \quad x = -1 \]
   
   c) The vertex is \( (-3, -4) \).
   \[ \frac{-b}{2a} = \frac{-6}{2(1)} = -3 \]
   \[ f(-3) = (-3)^2 + 6(-3) + 5 = 9 - 18 + 5 = -4 \]
   
   d) State the range of \( f \).
   \[ [-4, \infty) \]
   
   e) Graph. Label intercepts and vertex.

23. Given the polynomial \( g(x) = x^3 - x^2 + x - 6 \)
   
   a) State all possible rational zeros.
   \[ \pm \left\{ \frac{1, 2, 3, 6}{1} \right\} \]
   
   b) Find all zeros (real and complex.)
   
   \[ \frac{x}{1} - 1 - 1 - 6 \]
   \[ -1 \]
   \[ 1 \]
   \[ 2 \]
   \[ 0 \]
   \[ 2 \]
   \[ -4 \]
   \[ \text{not zero} \]
   
   \[ \frac{x}{1} - 1 - 1 - 6 \]
   \[ -1 \]
   \[ 2 \]
   \[ -3 \]
   \[ 1 \]
   \[ -2 \]
   \[ 3 \]
   \[ -9 \]

\[ \frac{2}{1} - 1 - 1 - 6 \]
\[ 2 \]
\[ 2 \]
\[ 2 \]
\[ 0 \leftrightarrow \text{zero}, \text{so } x = 2 \]
\[ x^2 + x + 3 = 0 \]
\[ x = \frac{-1 \pm \sqrt{1^2 - 4(1)(3)}}{2(1)} = \frac{-1 \pm \sqrt{-11}}{2} = \frac{-1 \pm i\sqrt{11}}{2} \]
\[ \text{Answer: } x = 2, x = \frac{-1 + i\sqrt{11}}{2}, x = \frac{-1 - i\sqrt{11}}{2} \]
Part III. There are 6 problems in this section. Choose any 4. Indicate in the boxes the problems you want graded. 10 points each. If you do not indicate which 4, the first 4 will be graded. No Extra Credit!

24. Graph the following function.

\[ f(x) = \begin{cases} 
-2x + 1, & x < 0 \\
-3, & x \geq 0 
\end{cases} \]

25. The graph to the right is a circle with center (5,1).

a) Find the length of the radius.

\[ r = \text{distance between } (5,1) \text{ and } (2,-3) \]

\[ = \sqrt{(5-2)^2 + (1+3)^2} \]

\[ = \sqrt{3^2 + 4^2} \]

\[ = \sqrt{9 + 16} = \sqrt{25} = 5 \]

b) State the equation of the circle in standard form.

\[ (x-5)^2 + (y-1)^2 = 25 \]

26. Given \( f(x) = 2x^2 + 4x \), find and simplify \( \frac{f(x+h) - f(x)}{h} \).

Note: \[ f(x+h) = 2(x+h)^2 + 4(x+h) \]

\[ = 2(x^2 + 2xh + h^2) + 4x + 4h \]

\[ = 2x^2 + 4xh + 2h^2 + 4x + 4h \]

\[ \frac{f(x+h) - f(x)}{h} = \frac{2x^2 + 4xh + 2h^2 + 4x + 4h - (2x^2 + 4x)}{h} \]

\[ = \frac{h(4x + 2h + 4)}{h} = 4x + 2h + 4 \]
27. Solve algebraically for \( x \).
\[
\log_2 x + \log_2(x - 2) = 3
\]
\[
\log_2 \left( \frac{x(x - 2)}{2} \right) = 3
\]
\[
2^3 = x^2 - 2x
\]
\[
8 = x^2 - 2x
\]
\[
0 = x^2 - 2x - 8
\]
\[
0 = (x - 4)(x + 2) \Rightarrow \begin{cases} x = 4 \setminus & x = -2 \end{cases}
\]

Check:
\[
x = 4 \quad \log_2(4) + \log_2(2) = \frac{3}{2} + 1 = 3
\]
\[
x = -2 \quad \log_2(-2) \text{ cannot be done}
\]

28. Solve \( \frac{x + 2}{x - 3} \geq 0 \). Express in interval form. To receive full credit you must show work that supports your answer.

VA:
\[
x - 3 = 0 \quad \Rightarrow x = 3
\]

Zero:
\[
x + 2 = 0 \quad \Rightarrow x = -2
\]

Interval:
\[
(-\infty, -2] \cup [3, \infty)
\]

Test Values:
\[
x = -3 \quad \frac{-3 + 2}{-3 - 3} = \frac{-1}{-6} = \frac{1}{6} \geq 0 \quad \text{True}
\]
\[
x = 0 \quad \frac{0 + 2}{0 - 3} = \frac{2}{-3} = -\frac{2}{3} \neq 0 \quad \text{False}
\]
\[
x = 4 \quad \frac{4 + 2}{4 - 3} = \frac{6}{1} = 6 \geq 0 \quad \text{True}
\]

29. Consider the following system:
\[
\begin{cases} 2x - 3y = -6 \\ x + y = 2 \end{cases}
\]

a) Solve algebraically.
Show all your work.
\[
\begin{align*}
2x - 3y &= -6 \\
3(x + y &= 2) \\
\rightarrow 3x + 3y &= 6 \\
5x &= 0 \\
x &= 0 \\
y &= 2
\end{align*}
\]
Answer: \((0, 2)\)

b) Solve graphically and explain how you obtained your answer by looking at the graph.