

[12]

1. EVALUATE EACH OF THE FOLLOWING. SHOW ALL WORK. SIMPLIFY YOUR ANSWERS TO A SINGLE INTEGER.

A) $6! - 4!$

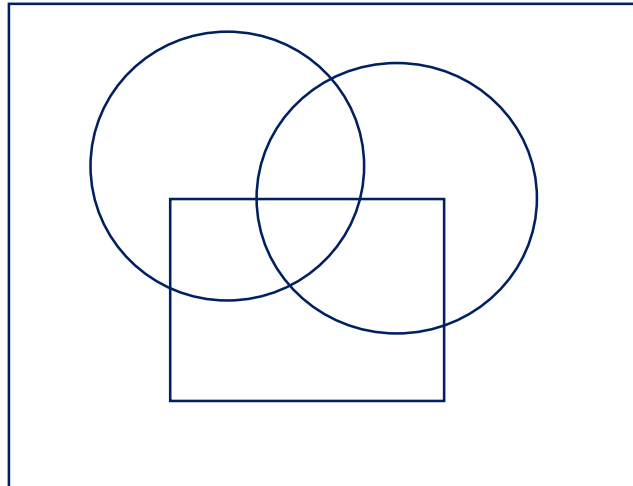
B) $C(7, 4)$

C) $P(9, 4)$

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2. In a recent survey of a senior class of 140 students the following information was found. Sixty-eight students took a math class, 62 took a chemistry class and 61 took a physics class. Thirty-three students took both math and chemistry, 32 took both physics and chemistry, 28 took math and physics, while 15 took all three subjects.

a) Complete the Venn diagram. Be sure to label each set.



b) How many students took only a math class?

c) How many students took none of the three subjects?

In problems 3, 4 and 5 you may leave your answers in terms of products, powers, permutations and/or combinations. You need not simplify your answers.

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3. A computer hacker has discovered that a personal password consist of three non-zero digits followed by 4 letters from the alphabet.

a) How many different passwords are possible?

b) How many different passwords are possible if no digit or letter is repeated?

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4. A basketball player makes 74% of all of her free throw attempts. If in a given game it is known she attempted 13 free throws.

a) Find the probability she makes exactly 9 free throws.

b) Find the probability she makes at least 11 free throws.

c) Find the probability she makes at least one free throw.

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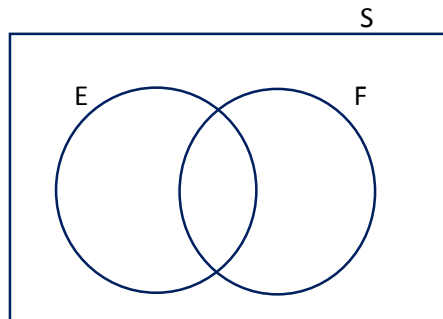
5. A hand of 6 cards is dealt from a regular 52 card deck. Find the probability that

- a) The hand contains all diamonds.
- b) The hand contains no queens or kings.
- c) The hand contains three jacks and at least 2 queens.

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6. Let E and F be events of sample space S . Let $P(E) = 0.6$, $P(F) = 0.3$ and the $P(E \cup F) = 0.7$.

a) Fill in all probabilities in the Venn diagram shown.



b) Find $P(E \cap \bar{F})$

c) Find $P(E \cap F)$

d) Find the $P(E | F)$

e) Are E and F independent events? Justify your answer.

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7. On a table are three identical boxes. Box A has 1 red and 2 green marbles, box B has 1 red and 1 green marble and box C has 1 red and 3 green marbles. A box is chosen and a marble is drawn.

a) Construct a tree diagram representing this problem. Label all branches and put on the correct probabilities.

b) What is the probability the marble drawn is green?

c) If the marble drawn is green what is the probability it came from the box containing the 4 marbles?

[10]

8. a) Find the equation of the line through the points $(4, 10)$ and $(-1, 4)$. Give your answer in slope intercept form.

b) Are the lines with equations $5x + 2y = 12$ and $y = \frac{-2}{5}x + 6$ parallel, intersecting, or coincident? Justify your answer.

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9. How many different arrangements to all of the letters in the word ALLELE? Show all work and give a single integer as your answer.

[8]

10. a) Let $D = \begin{pmatrix} 8 & 5 \\ 6 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 2 & 0 \\ 3 & 1 & 0 & 4 \end{pmatrix}$. Find the product DB .

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b) Let $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & -1 & 0 \\ 3 & 0 & 5 \end{pmatrix}$. Find A^{-1} the inverse of matrix A

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11. Solve the following system of equations using reduced row-echelon form. Label all row operations and clearly state your final solution.

$$2x + 5y + 4z = 20$$

$$x + 4y = 7$$

$$3x + y + 2z = 5$$

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12. The following augmented matrices represent systems of equations in terms of x , y , and z . For each case state the general solution or that no solution exists.

$$\text{a) } \left(\begin{array}{ccc|c} 0 & 0 & 0 & 5 \\ 1 & 0 & 1 & 12 \\ 0 & 1 & 0 & 0 \end{array} \right)$$

$$\text{b) } \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 2 & 8 \\ 0 & 3 & 0 & 6 \end{array} \right)$$

$$\text{c) } \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

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13. Formulate an LP model for the following problem. (DO NOT ATTEMPT TO SOLVE IT)

ACME manufacturing makes three types of road runner traps. Blue traps require two springs, 12 units of metal and four hours of labor. Red traps use only 10 units of metal, one more spring than blue traps and three hours of labor. Green traps require five springs, 13 units of metal, and two more hours of labor than a blue trap. Each week ACME has 3300 units of metal available and 1750 springs. In addition ACME has a labor contract with Coyotes Local Union 101 requiring that they will provide at least 20 hours of labor for each of eleven employees. Due to costs ACME cannot provide more than 40 hours of labor for each of its 25 employees each week. If the profit on green traps is \$35, for blue traps \$29 and \$16 for red traps, how many of each trap should ACME manufacture to maximize their profit.

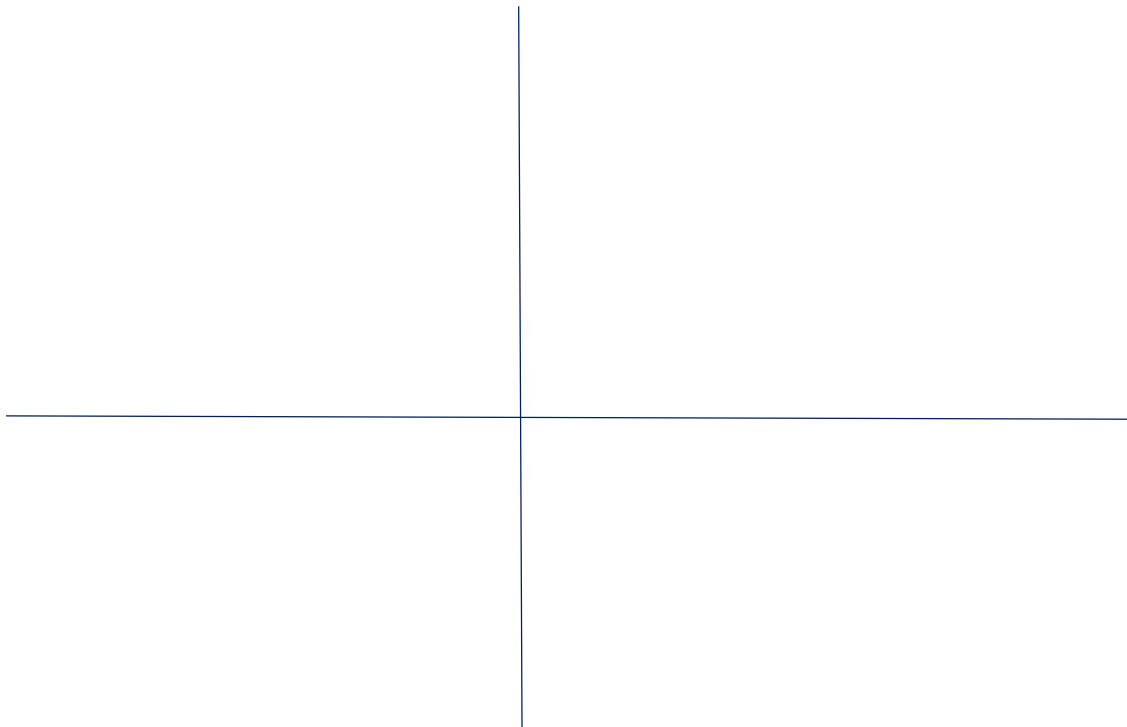
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14. Consider the following linear programming problem.

Maximize $P = 7x + 6y$ subject to the constraints:

$$x \geq 0, \quad y \geq 0, \quad x \leq 8, \quad 2x - 3y \geq -15, \quad 3x + 5y \leq 44$$

- a) Sketch the feasible region and label the corner points. Be sure to shade the feasible region.



- b) Solve the problem geometrically and report the complete solution.

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15. For each of the following tableaux, do one of the following.

- 1) If a pivot is required, write 'pivot needed', circle the pivot element BUT DO NOT PIVOT.
- 2) If there is no optimal solution say so and state why.
- 3) If the problem is finished report the complete solution.

a)

BV	P	x_1	x_2	s_1	s_2	RHS
x_2	0	1	-4	1	0	11
s_2	0	0	-2	-1	1	3
P	1	0	-1	3	0	19

b)

BV	P	x_1	x_2	x_3	s_1	s_2	s_3	RHS
s_1	0	1	-2	1	1	0	0	6
s_2	0	2	-1	3	0	1	0	14
s_3	0	4	0	-2	0	0	1	8
P	1	2	3	-2	0	0	0	0

c)

BV	P	x_1	x_2	x_3	s_1	s_2	s_3	RHS
x_1	0	1	4	2	1	0	0	11
x_2	0	0	3	5	0	2	1	13
s_3	0	0	2	1	0	1	1	7
P	1	0	3	2	1	2	0	12

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16. Use the simplex method to solve the following LP model. The number of pivots needed will be two or less.

Maximize $P = 5x_1 + 3x_2 + 2x_3$ subject to the constraints

$$2x_1 + 4x_2 + x_3 \leq 8$$

$$x_1 + 2x_2 + 3x_3 \leq 6$$

$$x_1 + x_2 + x_3 \leq 5$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$