

[12]

1. EVALUATE EACH OF THE FOLLOWING. SHOW ALL WORK. SIMPLIFY YOUR ANSWERS TO A SINGLE INTEGER.

A)  $5! + 4!$

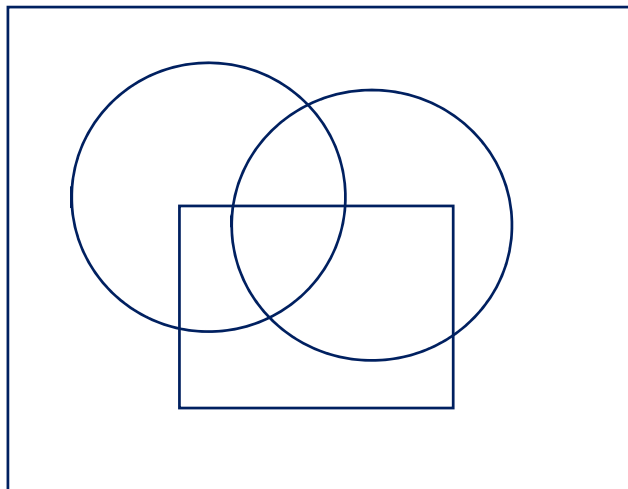
B)  $C(8, 3)$

C)  $P(10, 3)$

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2. In a recent survey of a senior class of 300 students the following information was found. One-hundred fifty six students took a math class, 147 took a chemistry class and 123 took a physics class. Sixty students took both math and chemistry, 63 took both physics and chemistry, 41 took math and physics, while 28 took all three subjects.

a) Complete the Venn diagram. Be sure to label each set.



b) How many students took only a chemistry class?

c) How many students took exactly two of the three subjects?

**In problems 3, 4 and 5 you may leave your answers in terms of products, powers, permutations and/or combinations. You need not simplify your answers.**

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3. A computer hacker has discovered that a personal password consist of two non-zero digits followed by 5 letters from the alphabet.

a) How many different passwords are possible?

b) How many different passwords are possible if no digit or letter is repeated?

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4. A basketball player makes 77% of all of her free throw attempts. If in a given game it is known she attempted 9 free throws.

a) Find the probability she made exactly 6 free throws.

b) Find the probability she made at least 7 free throws.

c) Find the probability she missed at least one free throw.

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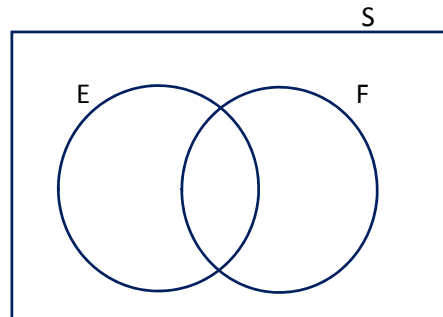
5. A hand of 5 cards is dealt from a regular 52 card deck. Find the probability that

- a) The hand contains all red cards.
  
  
  
  
  
  
  
  
  
  
- b) The hand contains no jacks, queens or kings.
  
  
  
  
  
  
  
  
  
  
- c) The hand contains two jacks and at least 1 queens.

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6. Let E and F be events of sample space S. Let  $P(E) = 0.7$ ,  $P(F) = 0.5$  and the  $P(E \cup F) = 0.9$ .

a) Fill in all probabilities in the Venn diagram shown.



b) Find  $P(E \cap \bar{F})$

c) Find  $P(E \cap F)$

d) Find the  $P(E | F)$

e) Are E and F independent events? Justify your answer.

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7. On a table are three identical boxes. Box A has 1 red and 2 green marbles, box B has 1 red and 1 green marble and box C has 2 red and 2 green marbles. A box is selected and a marble is drawn.

a) Construct a tree diagram representing this problem. Label all branches and put on the correct probabilities.

b) What is the probability the marble drawn is green?

c) If the marble drawn is green what is the probability it came from the box containing the 2 marbles?

[10]

8. a) Find the equation of the line through the points ( 9, 14 ) and ( -3, 6 ). Give your answer in slope intercept form.

b) Are the lines with equations  $4x + 7y = 21$  and  $y = \frac{-4}{7}x + 3$  parallel, intersecting, or coincident? Justify your answer.

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9. How many different arrangements to all of the letters in the word "EXCESSIVE" ? Show all work and give a single integer as your answer.

[8]

10. a) Let  $A = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}$  and  $B = \begin{pmatrix} 0 & 1 & 2 & 0 \\ 3 & 1 & 0 & 4 \end{pmatrix}$ . Find the product  $AB$ .

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b) Let  $A = \begin{pmatrix} 3 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$  Find  $A^{-1}$  the inverse of matrix  $A$

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11. Solve the following system of equations using reduced row-echelon form. Label all row operations and clearly state your final solution.

$$2x + 5y + 4z = 43$$

$$x + 4y = 16$$

$$3x + y + 2z = 25$$

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12. The following augmented matrices represent systems of equations in terms of  $x$ ,  $y$ , and  $z$ . For each case state the general solution or that no solution exists.

$$\text{a) } \left( \begin{array}{ccc|c} 0 & 0 & 1 & 7 \\ 1 & 2 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\text{b) } \left( \begin{array}{ccc|c} 2 & 0 & 0 & 6 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\text{c) } \left( \begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 4 \end{array} \right)$$

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13. Formulate an LP model for the following problem. ( DO NOT ATTEMPT TO SOLVE IT)

ACME manufacturing makes three types of road runner traps. Blue traps require two springs, 14 units of metal and four hours of labor. Red traps use only 10 units of metal, two more springs than blue traps and three hours of labor. Green traps require five springs, 11 units of metal, and one more hour of labor than a red trap. Each week ACME has 2300 units of metal available and 1950 springs. In addition ACME has a labor contract with Coyotes Local Union 101 requiring that they will provide at least 25 hours of labor for each of ten employees. Due to costs ACME cannot provide more than 40 hours of labor for each of its 30 employees each week. If the profit on green traps is \$55, for blue traps \$39 and \$19 for red traps, how many of each trap should ACME manufacture to maximize their profit?



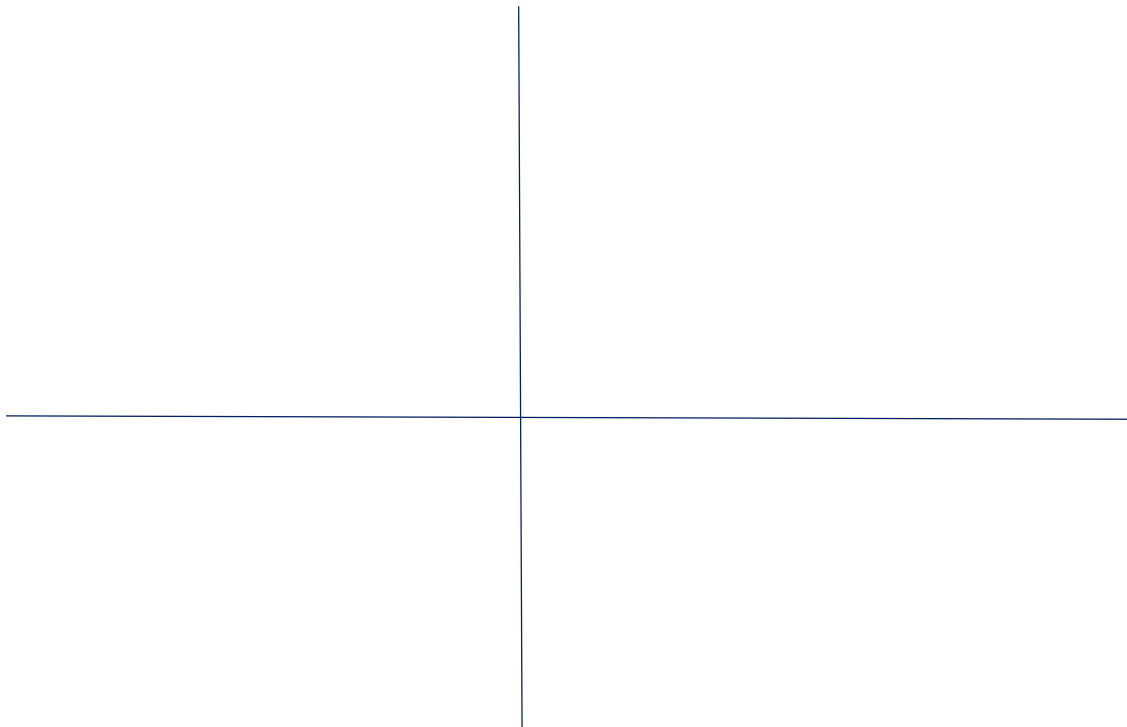
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14. Consider the following linear programming problem.

Maximize  $P = 6x + 9y$  subject to the constraints:

$$x \geq 0, \quad y \geq 0, \quad y \leq 8, \quad 8x - y \geq 16, \quad x + y \leq 13$$

- a) Sketch the feasible region and label the corner points. Be sure to shade the feasible region.



- b) Solve the problem geometrically and report the complete solution.

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15. For each of the following tableaux, do one of the following.

- 1) If a pivot is required, write 'pivot needed', circle the pivot element BUT DO NOT PIVOT.
- 2) If there is no optimal solution say so and state why.
- 3) If the problem is finished report the complete solution.

a)

BV	P	$x_1$	$x_2$	$s_1$	$s_2$	RHS
$x_1$	0	0	2	2	0	12
$s_2$	1	0	0	1	1	6
P	1	1	3	0	0	22

b)

BV	P	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	RHS
$s_1$	0	1	2	-1	1	0	0	6
$s_2$	0	2	1	-3	0	1	0	14
$s_3$	0	4	0	-2	0	0	1	8
P	1	2	3	-2	0	0	0	0

c)

BV	P	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	RHS
$x_1$	0	1	4	2	1	0	0	11
$x_2$	0	0	3	5	0	2	1	13
$s_3$	0	0	2	1	0	1	1	7
P	1	0	3	-2	1	2	0	12

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16. Use the simplex method to solve the following LP model. The number of pivots needed will be two or less.

Maximize  $P = 5x_1 + 3x_2 + 2x_3$  subject to the constraints

$$2x_1 + 4x_2 + x_3 \leq 16$$

$$x_1 + 2x_2 + 3x_3 \leq 12$$

$$x_1 + x_2 + x_3 \leq 10$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$