[12] points

1. Evaluate each of the following. SHOW ALL work and give your answer as a single integer.

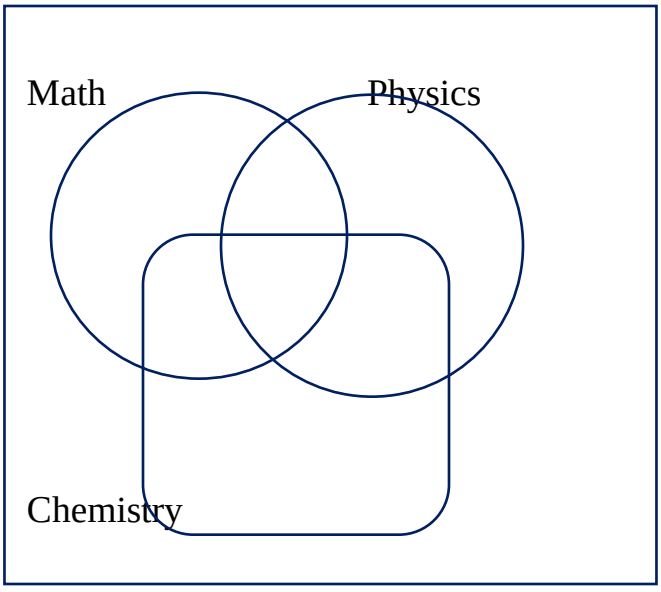
a) $\frac{12!}{10!} + 0! + 1! + 2!$

b) ${}_9P_3$

c) ${}_{10}C_3$

[10pts]

2. In a high school of exactly 250 students the following information is known: seventy-five students took a math class, sixty-six took a physics class and eighty eight took a chemistry class. In addition we know thirty took both math and physics, twenty-six took chemistry and physics, twenty-three took math and chemistry and a dozen took all three classes. Complete the Venn diagram and answer the following questions.



a) How many students took only a math class?

b) How many students took math and chemistry but not physics?

In problems 3,4 and 5 you may eave your answer in terms of products, powers, permutations and/or combinations. You need not simplify your answers.

[10pts]

3. At a university fraternities and sororities can be named using either two or three of the letters from the Greek alphabet, in which there are exactly 24 letters.

a) How many different names exist if no letter is allowed to repeat?

b) How many different names exist if repetition is allowed?

[15pts]

4. A professional golfer sinks a putt 85% of the time. If he plays a nine hole course and takes exactly one putt on each of the holes, what is the probability he

a) makes exactly six of them

b) makes at least seven of them

c) misses at least one of them

[12pts]

5. A red die and a green die each numbered one through six on their respective sides are rolled and the sum of the top two numbers is computed.

a) What is the probability the sum is exactly 5?

b) What is the probability the sum is at least seven?

c) What is the probability the sum is at least seven given the red die shows a three?

d) Given that the sum is at least seven, what is the probability the red die shows a three?

[8pts]

6. How many different arrangements are there of the all of the letters in the word "RECEED" ? Show all work and give a single integer as your answer.

[12pts]

7. A manufacturer gets two thirds of its parts from supplier A and the remainder from supplier B. Parts from supplier A are defective one-seventh of the time and from supplier B they are defective two-sevenths of the time.

a) Construct a tree diagram representing this problem.

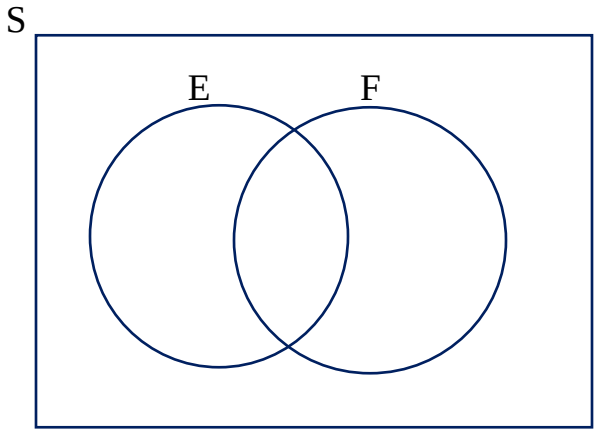
b) If a part is chosen at random what is the probability it is defective? Give your answer as an exact fraction.

c) If a chosen part is defective what is the probability it came from supplier A?

[12pts]

8. Let E and F be events of sample space S. Let P(E) = 0.3, P(E∪ F) =0.8 and P(F) = 0.7.

a) Complete the Venn diagram representing this information.



b) Find P(E ∩ F)

c) Find P (E | F ‘)

d) Find P (F ‘ | E)

e) Are E and F independent events? Justify your answer.

[8pts]

9. a) Are the lines with equations $2x + 3y = 12$ and $y = 2/3\ x + 4$ parallel, intersecting or coincident? Justify your answer.

b) Find the equation of the line through the point (2,5) which has the same slope as the line with equation $8x - 2y = 13$.

[8pts]

10. Let $A = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 0 & 4 \end{pmatrix}$ Find the product AB.

[12pts]

11. Let A = $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ find A^{-1} the inverse of A.

[12pts]

12. Solve the following using reduced row-echelon form. Label all row operations and clearly state your final solution.

$$2x + 3y + z = 6$$
$$3x + y + 4z = 3$$
$$x + 2z = 0$$

[12pts]

13. The following augmented matrices represent systems of equations in terms of x, y and z. For each one state the general solution or that no solution exists.

a) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 1 & 1 & 1 & 12 \\ 0 & 0 & 0 & 0 \end{array}\right)$

b) $\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \end{array}\right)$

c) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \end{array}\right)$

[15pts]

14. Formulate an LP model for the following problem. (DO NOT ATTEMPT TO SOLVE IT!!)

ACME makes road runner traps in three colors. The red traps require two springs, 22 ounces of metal, and 75 minutes of labor. Blue traps require three springs, 30 ounces of metal and 85 minutes of labor. The green traps require six less ounces of metal than blue traps, four springs and fifteen minutes less labor than red traps. Each week ACME has 350 springs available and 200 pounds of metal. Due to a labor contract with Coyote Local 101 Labor Union, ACME must provide at least 10 hours of labor to five employees and due to cost they cannot exceed 400 total hours of labor per week. ACME makes a profit of $33 on red traps, $26 on green traps and $31 dollars on blue traps. How many of each type of trap should ACME produce each week in order to maximize their profit?

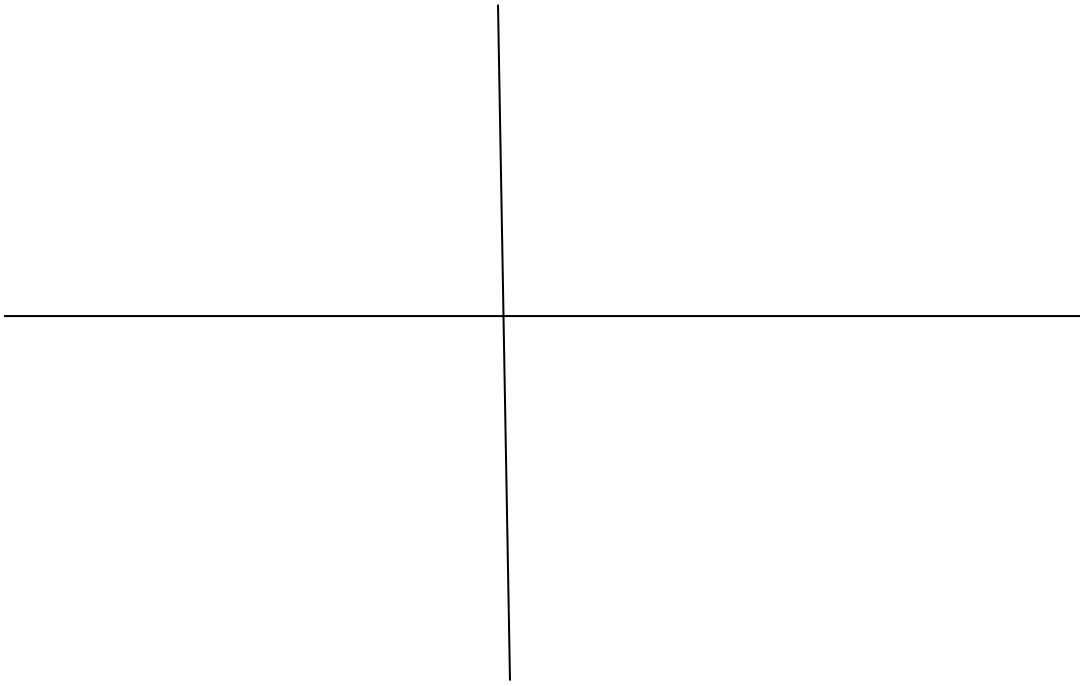
[15pts]

15. Consider the following linear programming problem.

Maximize $P = 8x + 9y$ subject to the constraints:

$$x \geq 0,\ \ y \geq 0,\ \ x + y \leq 11,\ \ 5x + 6y \leq 60$$

a) Sketch the feasible region and label the corner points



b) Solve the problem geometrically and report the complete solution.

[12pts]

16. For each of the following tableaux, do one of the following:

1. If a pivot is required, write ‘ pivot needed’, circle the pivot element, but DO NOT pivot.
2. If there is no optimal solution say so and state why.
3. If the problem is finished report the complete solution.

a)

BV	P	x_1	x_2	x_3	s_1	s_2	RHS
x_3	0	0	1	3	2	0	12
x_1	0	1	0	2	1	−2	3
P	1	0	0	5	1	1	19

b)

BV	P	x_1	x_2	s_1	s_2	RHS
s_1	0	1	2	1	0	24
s_2	0	1	3	2	1	33
P	1	2	−2	−1	1	44

c)

BV	P	x_1	x_2	x_3	s_1	s_2	s_3	RHS
x_1	0	1	1	0	2	1	1	15
s_2	0	0	1	−2	1	−1	1	12
s_3	0	0	1	−1	0	−1	2	−5
P	1	0	1	−1	1	2	1	27

[15pts]
17. Use the simplex method to solve the following LP problem. You will need two or less pivots.

Maximize: P = $5x_1 + 4x_2$ subject to the following constraints:

$x_1 \geq 0,\ x_2 \geq 0$, $x_1 + x_2 \leq 4$ and $2x_1 + 3x_2 \leq 9$.